Abstract

In this work, it is described how different synchronous machine models are derived; especially those used in the power system simulation software Simpow.

There are various ways how to describe the model of a synchronous machine, in the literature the biggest difference is the per unit system used in the models. Other differences are: the definitions of the $d$- and $q$-axis, the performance of the $dq0$-transform, and, of course, the number of damper windings modelled in the rotor circuit.

In this report, the per unit system used is thoroughly described, so the risk of misunderstanding is minimized.

Theoretical foundations have been found for the equations used in the FORTRAN-coded synchronous machine models implemented in Simpow. In this thesis, these equations are derived on a general basis, and are thoroughly derived for the models used in Simpow. There is also an explanation of relations between different models.

New DSL-coded models have been programmed for the four fundamental frequency models in Simpow. These new models are validated, and have been found to have a close resemblance with the “old” FORTRAN-models.
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1 Introduction

This report is one part of the result of the master thesis work in the Master of Science Electrical Engineering Programme, at the Royal Institute of Technology. The thesis was carried out at the department of Power Systems Analysis, ABB Utilities, in Västerås during the autumn 2001.

The other parts of the work are the code for the programmed machine models and a technical report for ABB.

1.1 Preface

Within Simpow, the power system simulation software developed by ABB, there are four synchronous machine models in the standard library of models. The differences between these models are the representation of the rotor circuits.

The models are coded using FORTRAN in the 1980’s, based on equations summarised in the technical reports [1] and [2]. The contents of the reports are mainly these equations, without any reference to the theory behind them. During the years, the models have been changed and these changes are only briefly documented. Additional documentation of the models is in the Simpow user’s manual [3], where the parameters that must be given to the model and the variables calculated by the models are listed.

The models have regularly been used in power system simulations and have been found to give enough accuracy for most cases. For other cases, special models have been made using the Simpow Dynamic Simulation Language, DSL. These are documented in special reports, [4] and [5].

1.2 Project description

- Using available theory and equations, realise the four synchronous machine models from Simpow with DSL.

- The realisation of the models is to be based on classical theory of the synchronous machine local reference system (two-axis theory, $dq0$-transformation, and equivalent schemes). Further, give the physical-mathematical description of the machine included in such global reference systems used when simulating power systems.

- The realised models shall by simulation of a sufficiently number of cases be validated against existing models. This will give opportunity to notice eventual imperfections.

- Theoretical foundation exists, but other literature should be examined. To start with, a fundamental frequency model shall be realised and validated, and, if there is time, so shall also an instantaneous value model.

- Documentation of the realised models, with references to appropriate literature, is a central part of the work. A clear description shall be made of the theory and of test cases used for validation.

- The work shall be documented in a technical report.

1.3 Outline

This report describes how different synchronous machine models are derived and gives a background to how different models are related.
Chapter 2 describes the basic definitions used in the report. In chapter 3 the derivation of the time independent and reciprocal machine equations, in SI units, is described for a general synchronous machine model. Whereas chapter 4 contains the per unit description of these equations. Chapter 5 thoroughly describes the four different machine models used in Simpow. In chapter 6, a short introduction to the DSL-programming and the DSL-coded models is given. While in chapter 7, these DSL-coded models are validated against the already existing FORTRAN-coded models. Chapter 8 consists of the conclusions drawn during the work and a summary over the work that is still to be done.

1.4 Acknowledgement

During this thesis work, I have had a lot of help from different people, but first and foremost from Mr. Jonas Persson, my 1st supervisor from the Department of Electrical Engineering at KTH. I wish to thank him for all the time he has spent with me, helping me to understand even the most simple of problems, even though he is busy with the fulfilment of his Technology Licentiate degree. I would also like to thank him for his positive thinking: ‘It is never too late to give up, Emil’, and his e-mail-mania that always is a source of joy.

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At the Power Systems Analysis department at ABB Utilities I would like to thank everybody for making me feel at home at the department, and for giving me the opportunity of attending the Simpow Basic Course, twice. Especially I would like to thank:

Mr. Lars Lindkvist, who has been most helpful with all my programming difficulties, even though I haven’t got a Simpow release named after me yet. And for his daily cheering comment: ‘Are you finished soon?’

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I would also like to thank my friend, Mr. Mathias Carlsson, for his comments on my work and the various conversations during Sunday dinner, after the Lützen-fog has left the kitchen.

At last, I would like to thank my family, for their support through all my years of study, and in particularly my mother who probably will be as relieved as I when this work is finished.
2 Theory

Throughout the report, upper-case letters are used when relating to quantities in SI units and lower-case when relating to per unit values. The exceptions are time, $t [s]$, speed, $\omega [rad/s]$, and angle, $\delta \phi \theta [rad], [degree]$. Bold letters are used when referring to a matrix or a vector. All symbols used in the report are defined in the List of symbols that is attached at the end of this report.

2.1 Physical description

To be able to understand different models of the synchronous machine, the properties of the machine must be understood. Here, the reader is assumed to have a basic knowledge of electrical machines, for further reading see e.g. [6] or [7]. To avoid misunderstandings, a short description including basic definition is included.

Fundamental frequency models are normally used when simulating the transient stability of larger power systems. In Simpow these models are used in the module called *Transta*. The instantaneous value models are used when simulating the detailed behaviour of particular machines, which in Simpow is made in the *Masta* module.

An electric machine is used for energy conversion. In a generator, mechanical energy is converted into electrical via electromagnetism, in a motor it is the other way around. There are no differences in the theoretical treatment of motor or generator, in this report generator definitions are used, i.e. the stator currents, $i_a$, $i_b$ and $i_c$, are defined positive out of the machine, see Figure 2.1.

![Figure 2.1 Definition of generator quantities and the direct- and quadrature axes](image)

**Figure 2.1 Definition of generator quantities and the direct- and quadrature axes**

2.1.1 Multiple pole machines

Synchronous machines are mainly operated at synchronous speed; i.e. the electrical frequency of the rotor is the same as the frequency of the stator (the net frequency).
The relationship between electrical frequency, $\omega$, and mechanical, $\omega_{\text{mech}}$, is the number of pole pairs.

$$\omega_{\text{mech}} = \frac{\omega}{\text{pole pairs}}$$ (2.1)

This means that the higher number of poles a machine has, the slower it rotates. Due to the symmetry of the machine, the same relation exists between electrical and mechanical degrees or radians. The symmetry makes it possible to model a multiple pole machine as a single pole pair machine.

### 2.1.2 Direct and quadrature axis

Two axes are defined, the direct axis, $d$-axis, and the quadrature axis, $q$-axis, see Figure 2.1. These axes are following the rotor; the $d$-axis is aligned with the magnetic flux vector generated by the field current, i.e. the magnetic north pole, and is lagging the $q$-axis by 90 electrical degrees. This is the most common definition, and is used by the IEEE Standard Dictionary of Electrical and Electronic Terms, [8].

### 2.2 Time dependency

Due to the non-uniform air gap, the reluctance of the magnetic path, $\mathcal{R}$, will be rotor position dependent, i.e. time dependent. This is due to the fact that:

$$\mathcal{R}(t) \propto l$$ (2.2)

Where $l$ represents the length of the magnetic path.

This effect is especially noticeable in a salient pole machine, but also in a round rotor machine. This also implies that the inductances of the stator circuits, as well as the rotor to stator mutual inductances, are time dependent, since:

$$L \propto \frac{1}{\mathcal{R}}$$ (2.3)

Where $L$ represents the inductances of the stator circuits.

For the present modelling work, the structure of the stator will be assumed to be perfectly smooth, with evenly distributed windings, this will cause no position dependency of the rotor inductances [9]. That is, from the rotor point of view, the air gap is not time dependent.

### 2.2.1 The dq0-transformation

To be able to get a time independent equation system, the stator quantities (voltages, currents and flux linkages) are transformed using the $dq0$-transformation. This transformation, sometimes called the Park- or the Blondel transformation, is a transformation to rotor coordinates.

Bühler, [10], has thoroughly described how the $dq0$-transformation can be divided into two steps, see Figure 2.2.
2 Theory

Time dependency

a) synchronous machine with the three-phase stator windings

b) synchronous machine with the three-phase stator windings transformed to the two-phase global reference system

c) synchronous machine with the two-phase global reference windings transformed to the two-phase local reference system

Figure 2.2 Schematic view of a two-pole three-phase synchronous machine
Detailed Description of Synchronous Machine Models Used in Simpow

2.2.1.1 Three-phase stator to two-phase global transform

Assuming that the global reference axis coincides with the \( a \)-axis of the machine, the first step is a transformation of the three-phase stator quantities, indices \( a, b \) and \( c \), to a two-phase global reference system, indices \( \alpha \) and \( \beta \), see Figure 2.2b.

\[
U_{a\beta} = U_a + jU_{\beta} = k [U_a + aU_b + a^2U_c]
\]  

(2.4)

Where \( k \) is an arbitrary transformation constant and \( a = e^{-j\gamma/3} \) 

(2.5)

If \( k \) is chosen as \( k = \gamma_0 \), a peak value invariant transformation is obtained.

\[
U_{a\beta} = \frac{2}{3} [U_a + aU_b + a^2U_c]
\]  

(2.6)

Now \( U_a \) and \( U_{\beta} \) in equation 2.4 becomes:

\[
U_{a} = \frac{2}{3} [U_a - \frac{1}{2}U_b - \frac{1}{2}U_c]
\]  

(2.7)

\[
U_{\beta} = \frac{1}{\sqrt{3}} [U_b - U_c]
\]  

(2.8)

Other choices of \( k \) can be made; a common choice is \( k = \sqrt[3]{2} \), which makes the transform power invariant. Different advantages and disadvantages with these transformations are described in [9].

2.2.1.2 Two-phase global to local transform

In the second step, the time dependency is extracted from the equation system. The two-phase global reference quantities are transformed to the local rotor coordinate system, indices \( d \) and \( q \), see Figure 2.2c.

\[
U_{dq} = U_d + jU_q = U_{a\beta}e^{-j\theta}
\]  

(2.9)

Here \( \theta \) is the electrical displacement angle between the real-axis of the global reference frame and the real axis of the local system, i.e. the \( d \)-axis of the rotor, see Figure 2.1. Separating the real and imaginary parts, the \( d \)- and \( q \)-axis quantities are described as:

\[
U_d = [U_a \cos \theta + U_b \sin \theta]
\]  

(2.10)

\[
U_q = [-U_a \sin \theta + U_b \cos \theta]
\]  

(2.11)

2.2.1.3 Final dq0-transform

Together, these two steps gives, after some trigonometric calculations:

\[
\begin{bmatrix}
U_d \\
U_q \\
U_0
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
\cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\
-\sin(\theta) & -\sin(\theta - \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix} \begin{bmatrix}
U_a \\
U_b \\
U_c
\end{bmatrix}
\]  

(2.15)

Now, the final \( dq0 \)-transformation looks like:
or shorter:
\[ U_{dq0} = BU_S \]  
(2.16)

Where,

- \( U_{dq0} \) are the stator voltages in the local rotor reference system.
- \( U_S \) are the three-phase stator voltages.
- \( B \) is the transformation matrix.

Now, the \( dq0 \)-transformation has been defined for the stator voltages, but the same is valid for the stator currents and flux linkages, see Equations (2.17) and (2.18).

\[ \Psi_{dq0} = B\Psi_S \]  
(2.18)

The inverse transform is given by:

\[
U_S = \begin{bmatrix}
\cos(\theta) & -\sin(\theta) & 1 \\
\cos(\theta - \frac{\pi}{2}) & -\sin(\theta - \frac{\pi}{2}) & 1 \\
\cos(\theta + \frac{\pi}{2}) & -\sin(\theta + \frac{\pi}{2}) & 1 \\
\end{bmatrix}
\]

\[ U_{dq0} = B^{-1}U_{dq0} \]  
(2.19)

\[ 2.3 \quad \text{Reference systems in Simpow} \]

Every synchronous machine has its own local reference system, in which its armature currents and voltages are expressed. In the network, these quantities belong to a common global reference system. Therefore, transformations have to be made between these reference systems in every node where a synchronous machine is connected.

The models described in this report are transforming the voltages from the global to the local system, where they are used when calculating the machine characteristic variables: \( \delta \) or \( \theta \), \( \omega \), \( \Psi_d \), \( \Psi_q \), \( \Psi_{fd} \), \( \Psi_{kq} \), \( I_d \), \( I_q \), \( \Psi_{ap} \), \( L_{d6} \), \( L_{q6} \), \( L_{k6} \), \( L_{kq} \), \( T_m \), \( T_e \), \( P \) and \( Q \); which all will be discussed in this report. After the currents out of the machine, \( I_d \), \( I_q \), are calculated, they are transformed back to the global reference system.

As described in the previous sub-chapter, the transformation of the voltages from the global- to a local reference system looks like:

\[ U_{dq} = U_d + jU_q = U_{ap}e^{-j\phi} \]  
(2.20)

This is valid when \( \phi \) is the electrical displacement angle between the real-axis of the global reference frame and the real axis of the local system. In Simpow the global reference system is defined in different ways depending on whether a fundamental frequency model or an instantaneous value model is made.

\[ 2.3.1 \quad \text{Fundamental frequency models} \]

For a fundamental frequency model, the reference frame is a complex plane rotating with the system frequency \( \omega_n \) [rad/sec]; here the electrical displacement angle \( \delta \) is measured between the real-axis of the reference frame and the \( q \)-axis of the rotor. This changes the transformation equation (2.9), to:

\[ U_{dq} = U_{ap}e^{-j(\delta - \frac{\pi}{2})} \]  
(2.21)
and the \( d \)- and \( q \)-axis quantities are described as:

\[
U_d = [U_a \sin \delta - U_b \cos \delta]
\]

\[ \tag{2.22} \]

\[
U_q = [U_a \cos \delta + U_b \sin \delta]
\]

\[ \tag{2.23} \]

For the inverse transform, the \( \alpha \)- and \( \beta \)-quantities looks like:

\[
U_\alpha = [U_d \sin \delta + U_q \cos \delta]
\]

\[ \tag{2.24} \]

\[
U_\beta = [-U_d \cos \delta + U_q \sin \delta]
\]

\[ \tag{2.25} \]

In Simpow, the electrical displacement angle \( \delta \) is defined as Equation (2.26), [4].

\[
\delta = \phi_{\delta 0} + \delta_{\alpha 0} + \delta_{\text{dev}}
\]

\[ \tag{2.26} \]

Where

- \( \phi_{\delta 0} \) is the machine-node voltage angle, calculated in a power-flow
- \( \delta_{\alpha 0} \) is the internal load angle of the machine at time \( t = 0 \)
- \( \delta_{\text{dev}} \) is the integrated deviation angle, and is calculated as:

\[
\delta_{\text{dev}} = \int_0^t (\omega - \omega_{\delta}) dt
\]

\[ \tag{2.27} \]

\( \omega \) is the angular frequency of the machine

### 2.3.2 Instantaneous value models

For an instantaneous value model, the reference frame is the \( dq \)-axes of a reference machine rotating with the angular frequency \( \omega_{\text{ref}} \). Here the electrical displacement angle \( \theta \) is measured between the \( d \)-axis of the reference machine and the \( d \)-axis of the present machine, i.e. between the real-axis of the global reference frame and the real axis of the local system. This means that no changes are made in the transformation equation (2.9), and the \( d \)- and \( q \)-axis quantities are described as:

\[
U_d = [U_a \cos \theta + U_b \sin \theta]
\]

\[ \tag{2.28} \]

\[
U_q = [-U_a \sin \theta + U_b \cos \theta]
\]

\[ \tag{2.29} \]

For the inverse transform, the \( \alpha \)- and \( \beta \)-quantities looks like:

\[
U_\alpha = [U_d \cos \theta - U_q \sin \theta]
\]

\[ \tag{2.30} \]

\[
U_\beta = [U_d \sin \theta + U_q \cos \theta]
\]

\[ \tag{2.31} \]

In Simpow, the electrical displacement angle \( \theta \) is defined as Equation (2.32), [4].

\[
\theta = \delta - (\phi_{\text{ref}} + \delta_{\text{ref}} + \delta_{\text{dev}})
\]

\[ \tag{2.32} \]

Where

- \( \delta \) is defined in Equation (2.26)
- \( \phi_{\text{ref}} \) is the reference machine-node voltage angle, calculated in a power-flow.
- \( \delta_{\text{ref}} \) is the internal load angle of the reference machine at time \( t = 0 \).
- \( \delta_{\text{dev}} \) is the integrated deviation angle of the reference machine, and is calculated as:
Reference systems in Simpow

\[ \delta_{\text{ref dev}} = \int_0^t (\omega_{\text{ref}} - \omega_n) \, dt \quad (2.33) \]

\( \omega_{\text{ref}} \) is the angular frequency of the reference machine.

### 2.3.3 Conclusions

In Simpow, the global to local transformation looks like:

\[
U_d = [U_{\alpha} \sin \delta - U_{\beta} \cos \delta] = [U_{\alpha} \cos \theta + U_{\beta} \sin \theta] \quad (2.34)
\]

\[
U_q = [U_{\alpha} \cos \delta + U_{\beta} \sin \delta] = [-U_{\alpha} \sin \theta + U_{\beta} \cos \theta] \quad (2.35)
\]

and the local to global transformation looks like:

\[
U_{\alpha} = [U_d \sin \delta + U_q \cos \delta] = [U_d \cos \theta - U_q \sin \theta] \quad (2.36)
\]

\[
U_{\beta} = [-U_d \cos \delta + U_q \sin \delta] = [U_d \sin \theta + U_q \cos \theta] \quad (2.37)
\]

Where

\( \theta \) is the electrical displacement angle for an instantaneous value model, and is measured between the \( d \)-axis of the reference machine and the \( d \)-axis of the present machine.

\( \delta \) is the electrical displacement angle for a fundamental frequency model, and is measured between the real-axis of the reference frame and the \( q \)-axis of the rotor.

Even though \( \theta \) does not have to be equal to \( \delta - \pi/2 \), the equations in the \( dq0 \)-transformation can be treated in this way. Therefore, in the \( dq0 \)-transformation, the only difference between the fundamental frequency models and the instant value models is how the reference angle is calculated. This is why, in the following equations, only the angle \( \theta \) will be used.

It should be noted that there are often different per unit systems for the machine and for the system, this must be taken under consideration, when transforming between the global and local reference systems. This matter will be further discussed in chapter 4.4.
3 Mathematical description

In this chapter, a general description of the mathematical theory of the synchronous machine models is made. That is, the equations derived in this chapter are valid for the instantaneous value models, while for the fundamental frequency models some changes have to be made. These changes are discussed in chapter 1.

3.1 Machine equations

The performance of a machine can be described by the machine equations (bold letters are used when referring to matrices or vectors).

\[
\Psi = L \cdot I \quad (3.1)
\]

\[
U = -R \cdot I + \frac{d}{dt} \Psi \quad (3.2)
\]

\[
J \frac{d}{dt} \omega_{mech} = M_m - M_e \quad (3.3)
\]

Where

\[\Psi\] is the flux linkage matrix of the machine.

\[L\] is the inductance matrix of the machine.

\[I\] is the current matrix of the machine.

\[U\] is the voltage matrix of the machine.

\[R\] is the resistance matrix of the machine.

\[J\] is the combined moment of inertia of machine and turbine/load.

\[\omega_{mech}\] is the rotor speed, in mechanical radians per second.

\[M_m\] is the mechanical torque applied on the machine axis.

\[M_e\] is the electrical torque produced/consumed by the machine.

For a general machine, with one field winding, \(k\) damper windings in the d-axis and \(m\) damper windings in the q-axis, the flux linkage, inductance, current, voltage and resistance matrices look like:

\[
\Psi = \begin{bmatrix} \Psi_a & \Psi_b & \Psi_c & \Psi_f & \Psi_{kd} & \Psi_{mq} \end{bmatrix}^T = [\Psi_S \quad \Psi_R]^T \quad (3.4)
\]

\[
L = \begin{bmatrix}
-L_{aa} & -L_{ab} & -L_{ac} & L_{af} & L_{akd} & L_{amq} \\
-L_{ab} & -L_{bb} & -L_{bc} & L_{bf} & L_{bkd} & L_{bmq} \\
-L_{ac} & -L_{bc} & -L_{cc} & L_{cf} & L_{ckd} & L_{cmq} \\
L_{af} & L_{bf} & L_{cf} & L_f & L_{fkd} & L_{fkd} & L_{kd} & 0 \\
L_{akd} & L_{bkd} & L_{ckd} & L_{fkd} & L_{kd} & 0 \\
L_{amq} & L_{bmq} & L_{cmq} & 0 & 0 & L_{mq} \\
\end{bmatrix}
\]

\[
I = \begin{bmatrix} I_a & I_b & I_c & I_f & I_{kd} & I_{mq} \end{bmatrix}^T = [I_S \quad I_R]^T \quad (3.5)
\]

\[
U = \begin{bmatrix} U_a & U_b & U_c & U_f & 0 & 0 \end{bmatrix}^T = [U_S \quad U_R]^T \quad (3.6)
\]
In addition, $M_e$ can be described as:

$$M_e = \frac{3}{2} \frac{\omega_{\text{mech}}}{\omega} \text{Im} \{ \Psi_S^* I_S \} \quad (3.9)$$

Where the indices are:

- $a$, $b$, $c$ and $S$ are referring to the three phase stator related quantities
- $f$, $kd$, $mq$ and $R$ are referring to the two-phase rotor related quantities
- $f$ is referring to field winding related quantities
- $kd$ is referring to quantities related to the $k$ damper windings of the $d$-axis
- $mq$ is referring to quantities related to the $m$ damper windings of the $q$-axis

Thus,

- $\Psi_a$, $\Psi_b$, $\Psi_c$ and $\Psi_S$ are the flux linkages of the stator windings $a$, $b$ and $c$
- $\Psi_S^*$ is the complex conjugate of the stator flux linkages
- $I_a$, $I_b$, $I_c$ and $I_S$ are the currents in the stator windings $a$, $b$ and $c$
- $U_a$, $U_b$, $U_c$ and $U_S$ are the voltages over the stator windings $a$, $b$ and $c$
- $\Psi_f$, $\Psi_{kd}$, $\Psi_{mq}$ are the flux linkages of the rotor windings $f$, $kd$ and $mq$
- $I_f$, $I_{kd}$, $I_{mq}$ are the currents in the rotor windings $f$, $kd$ and $mq$
- $U_f$, $U_{kd}$, $U_{mq}$ are the voltages over the rotor windings $f$, $kd$ and $mq$
- $R_a$ is the armature resistance
- $R_f$ is the resistance of the field winding
- $R_{kd}$ is the resistance of the $d$-axis damper windings
- $R_{mq}$ is the resistance of the $q$-axis damper windings
- $\omega$ is the rotor speed, in electrical radians per second
- $L_{SS}$ is containing the stator self and mutual inductances
- $L_{SR}$ is containing the stator to rotor mutual inductances
- $L_{RR}$ is containing the rotor self and mutual inductances

As described in chapter 2.2, the matrixes $L_{SS}$ and $L_{SR}$ are time dependent.

Index $k$ and $m$ refers to the $k$ and the $m$ damper windings of the $d$- resp. the $q$-axis. E.g. if there are 1 damper winding in the $d$-axis and 2 damper windings in the $q$-axis, the vector $I_R$ and the matrix $L_{RR}$ would look like:

$$I_R = [I_f \ I_{kd} \ I_{mq}]^T = [I_f \ I_d \ I_{kd} \ I_{mq}]^T$$

$$R = \begin{bmatrix}
R_a & 0 & 0 & 0 & 0 \\
0 & R_a & 0 & 0 & 0 \\
0 & 0 & R_a & 0 & 0 \\
0 & 0 & 0 & -R_f & 0 \\
0 & 0 & 0 & 0 & -R_{kd} \\
0 & 0 & 0 & 0 & 0 & -R_{mq} \\
\end{bmatrix} = \begin{bmatrix}
R_S \\
- R_R \\
\end{bmatrix} \quad (3.8)$$

$$\Psi_{mech} = \frac{3}{2} \frac{\omega}{\omega} \text{Im} \{ \Psi_S^* I_S \}$$
3.2 Time dependency

The self-inductances of the stator windings, \( L_{aa}, L_{bb} \) and \( L_{cc} \), are assumed to have the same maximum value, \( L_{aa} \). These inductances can be divided into one constant term, \( L_{aa0} \), and one term dependent of the second and higher order harmonics, see Equation (3.10). The terms with higher order harmonics can often be neglected [9].

\[
L_{aa} = L_{aa0} + \sum_{n>1} L_{aan} \cos(n\theta) = L_{aa0} + L_{aan2} \cos 2\theta
\]  

(3.10)

Where

- \( L_{aa0} \) is the constant term of the self-inductances of the stator.
- \( L_{aan2} \) is the maximum value of the term that is dependent of the second order harmonic.
- \( L_{aan} \) is the maximum value of the term that is dependent of the \( n \)th order harmonic.

\( \theta \) is the electrical displacement angle for the machine.

In the same way, the mutual inductances between the stator windings, \( L_{ab}, L_{ac} \) and \( L_{bc} \), are assumed to have the same maximum value, \( L_{ab} \). These can be divided into one constant term, \( L_{ab0}, \) and one term dependent of the second order harmonics [9]. Due to the distribution of the windings, a 60-degree phase displacement will occur.

\[
L_{ab} = -L_{ba} = -L_{ab0} + L_{ab2} \cos(2\theta + \frac{\pi}{3}) = -L_{ab0} - L_{ab2} \cos 2(\theta - \frac{\pi}{6})
\]  

(3.11)

Where

- \( L_{ab0} \) is the constant term of the mutual inductances between the stator windings.
- \( L_{ab2} \) is the maximum value of the term that is dependent of the second order harmonic.

The maximum value of the mutual inductances between the stator and the rotor, \( L_{df}, L_{df}, L_{qf}, L_{d0}, L_{d0}, L_{s0}, L_{mq}, L_{mq}, L_{mq}, L_{mq} \), are assumed to be independent of which stator winding that is in question, and are called: \( L_{df}, L_{d0}, \) and \( L_{mq} \). These are dependent of the fundamental frequency, with a 90-degree lead of the mutual inductances between the stator and the \( q \)-axis windings of the rotor [9].

\[
L_{df} = L_{df} \cos \theta
\]  

(3.12)

\[
L_{d0} = L_{d0} \cos \theta
\]  

(3.13)

\[
L_{mq} = L_{mq} \sin \theta
\]  

(3.14)

Where \( L_{df}, L_{d0}, \) and \( L_{mq} \) are the mutual inductances between the rotor windings and the fictive, \( dq \) transformed, local stator windings \( d \) and \( q \).

The time dependent matrixes \( L_{SS} \) and \( L_{SR} \) can now be described as:

\[
L_{RR} = \begin{bmatrix} L_{dd} & 0 \\ 0 & L_{mq} \end{bmatrix} = \begin{bmatrix} L_{dd} & 0 & 0 \\ 0 & L_{dq} & L_{12q} \\ 0 & L_{12q} & L_{2q} \end{bmatrix}
\]
$L_{SS} = \begin{bmatrix} L_{aa0} & -L_{ab0} & -L_{ab0} \\ -L_{ab0} & L_{aa0} & -L_{ab0} \\ -L_{ab0} & -L_{ab0} & L_{aa0} \end{bmatrix} + \\
\begin{bmatrix} L_{aa2} \cos(2\theta) & -L_{ab2} \cos(2\theta + \frac{1}{2}\pi) & -L_{ab2} \cos(2\theta - \frac{1}{2}\pi) \\ -L_{ab2} \cos(2\theta + \frac{1}{2}\pi) & L_{aa2} \cos(2\theta - \frac{1}{2}\pi) & -L_{ab2} \cos(2\theta - \pi) \\ -L_{ab2} \cos(2\theta - \frac{1}{2}\pi) & -L_{ab2} \cos(2\theta - \pi) & L_{aa2} \cos(2\theta + \frac{1}{2}\pi) \end{bmatrix}$

$\begin{bmatrix} \cos(\theta) & \cos(\theta - \frac{1}{2}\pi) & \sin(\theta) \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} U_S = B U_{dq0}$

$U_{dq0} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 1 \\ \cos(\theta - \frac{1}{2}\pi) & -\sin(\theta - \frac{1}{2}\pi) & 1 \\ \cos(\theta + \frac{1}{2}\pi) & -\sin(\theta + \frac{1}{2}\pi) & 1 \end{bmatrix} U_{dq0} = B^{-1} U_{dq0}$

As defined earlier, the index $dq0$ refers to the stator quantities in the local rotor reference system, i.e. the $d$, $q$ and 0 indices refers to the fictive $d$-, $q$- and 0-windings. Inserting the transformed components into the expressions for the stator variables, the following expressions are reached after some reduction of trigonometric terms.

$M_e = \frac{3}{2} \frac{\omega_{mech}}{\omega} (\Psi_q I_d - \Psi_d I_q)$

Where

$L_d = L_{aa0} + L_{ab0} + \frac{1}{2} L_{aa2}$

$L_q = L_{aa0} + L_{ab0} - \frac{1}{2} L_{aa2}$

$L_0 = L_{aa0} - 2L_{ab0}$

$L_d$, $L_q$ and $L_0$, are the self-inductances of the fictive $d$-, $q$- and 0-windings respectively.

In the voltage equations, the last term is called the speed voltage, and is a consequence of the transformation from a stationary to a rotating reference frame. These speed voltages are the dominant components in the voltage equations.
The terms $\frac{d}{dt}\psi_d$ and $\frac{d}{dt}\psi_q$, are called the transformer voltages, and can often be neglected during transient conditions [9].

The speed of the machine, $\omega$, is equal to the time derivative of the angle $\theta$.

In the rotor flux linkage equations, the transformed currents substitute the stator currents and the following relations are calculated for the rotor windings.

$$\Psi_R = \begin{bmatrix} \Psi_f \\ \Psi_{kd} \\ \Psi_{mq} \end{bmatrix} = \begin{bmatrix} -\frac{3}{2}L_{df} & 0 & 0 & L_f & L_{phd} & 0 \\ -\frac{3}{2}L_{dkd} & 0 & 0 & L_{phd} & L_{kd} & 0 \\ 0 & -\frac{3}{2}L_{qmq} & 0 & 0 & 0 & L_{mq} \end{bmatrix} \begin{bmatrix} I_F \\ I_R \end{bmatrix}$$

(3.25)

$$U_R = R_R I_R + \frac{d}{dt}\Psi_R$$

(3.26)

It is obvious that the $dq0$-transformation results in an equation system where all the inductances are independent of the rotor position. This makes the equations easier to solve; i.e. it makes the simulation faster.

### 3.4 Reciprocity

Although the $dq0$-transformation results in constant inductances, the mutual inductance coefficients between rotor and stator are non-reciprocal.

$$\frac{\partial \Psi_d}{\partial I_f} = L_{df}, \text{ but } \frac{\partial \Psi_f}{\partial I_d} = \frac{3}{2}L_{df}.$$  

An essential condition for the existence of a static equivalent circuit is the reciprocity of the mutual inductances [11].

This difficulty arises because a peak value invariant $dq0$-transformation was chosen. It could easily have been avoided, for example by choosing a power invariant transformation. The problem can be solved in different ways, either by using a reciprocal per unit system, see e.g. [9], or by simply changing the rotor currents by a factor $\frac{3}{2}$, see e.g. [11], here the latter is chosen. Defining the reciprocal stator-rotor mutual inductances as:

$$L_{df}' = \frac{1}{2}L_{df}$$

(3.28)

$$L_{dkd}' = \frac{3}{2}L_{dkd}$$

(3.29)

$$L_{qmq}' = \frac{3}{2}L_{qmq}$$

(3.30)

the reciprocal rotor self inductances and resistances as:
\[ L_{f'} = \frac{\gamma}{2} L_f \]  
\[ L_{jkd'} = \frac{\gamma}{2} L_{jkd} \]  
\[ L_{jmq'} = \frac{\gamma}{2} L_{jmq} \]  
\[ R_{f'} = \frac{\gamma}{2} R_f \]  
\[ R_{kd'} = \frac{\gamma}{2} R_{kd} \]  
\[ R_{mq'} = \frac{\gamma}{2} R_{mq} \]  

and the reciprocal rotor currents as:

\[ I_{f'} = \frac{\gamma}{2} I_f \]  
\[ I_{kd'} = \frac{\gamma}{2} I_{kd} \]  
\[ I_{mq'} = \frac{\gamma}{2} I_{mq} \]

Inserting these reciprocal quantities, in the flux linkage Equation (3.27), gives:

\[
\begin{bmatrix}
\Psi_d' \\
\Psi_q' \\
\Psi_{kd}' \\
\Psi_{mq}'
\end{bmatrix} =
\begin{bmatrix}
-L_{d} & 0 & 0 & L_{df'} & L_{dkd'} & 0 \\
0 & -L_{q} & 0 & 0 & 0 & L_{dqq'} \\
0 & 0 & -L_{0} & 0 & 0 & 0 \\
-L_{kd} & 0 & 0 & L_{jkd} & L_{kd} & 0 \\
0 & -L_{dmq} & 0 & 0 & 0 & L_{mq}
\end{bmatrix}
\begin{bmatrix}
I_d' \\
I_q' \\
I_{0}' \\
I_{kd}' \\
I_{mq}'
\end{bmatrix}
\]

(3.40)

It is now obvious that the mutual inductance coefficients between rotor and stator are reciprocal. Since there is no reason to use the non-reciprocal values of the stator-rotor mutual inductances, the rotor self inductances and resistances or the rotor currents, the primes in Equation (3.40) will be omitted in the following calculations of this report.

Dividing the matrix into the \(d\)-axis and the \(q\)-axis flux linkages, and including the effects of the leakage inductances, the flux linkage equations can be rewritten as:

\[
\begin{bmatrix}
\Psi_d \\
\Psi_q \\
\Psi_{kd} \\
\Psi_{mq}
\end{bmatrix} =
\begin{bmatrix}
-L\epsilon_d & L_{df} & L_{dkd} & 0 \\
0 & -L\epsilon_q & 0 & L_{dqq} \\
0 & 0 & -L\epsilon_0 & 0 \\
-L\epsilon_{kd} & 0 & 0 & L_{jkd}
\end{bmatrix}
\begin{bmatrix}
L_{d} & L_{df} & L_{dkd} & 0 \\
L_{df} & L_f & 0 & L_{fkd} \\
L_{dkd} & L_{fkd} & (L_k - L_{ckd}) + L_{ckd} & 0 \\
0 & 0 & 0 & L_{mq}
\end{bmatrix}
\begin{bmatrix}
I_d \\
I_q \\
I_{0} \\
I_{kd} \\
I_{mq}
\end{bmatrix}
\]

(3.41)

\[
\begin{bmatrix}
\Psi_d \\
\Psi_q \\
\Psi_{kd} \\
\Psi_{mq}
\end{bmatrix} =
\begin{bmatrix}
-L\epsilon_d & L_{df} & L_{dkd} & 0 \\
0 & -L\epsilon_q & 0 & L_{dqq} \\
0 & 0 & -L\epsilon_0 & 0 \\
-L\epsilon_{kd} & 0 & 0 & L_{jkd}
\end{bmatrix}
\begin{bmatrix}
L_{df} & L_{dqm} & L_{dqm} & L_{dqm} \\
L_{dqm} & L_{jkd} & L_{jkd} & L_{jkd} \\
L_{dqm} & L_{jkd} & L_{jkd} & L_{jkd} \\
0 & 0 & 0 & L_{mq}
\end{bmatrix}
\begin{bmatrix}
I_d \\
I_q \\
I_{kd} \\
I_{mq}
\end{bmatrix}
\]

(3.42)

Where

- \(L_{\epsilon_d}\) is the leakage inductance of the \(d\)-winding
- \(L_{\epsilon_q}\) is the leakage inductance of the field winding
- \(L_{\epsilon_{kd}}\) is the leakage inductance of the damper windings in the \(d\)-axis
- \(L_{\epsilon_q}\) is the leakage inductance of the \(q\)-winding
- \(L_{\epsilon_{mq}}\) is the leakage inductance of the damper windings in the \(q\)-axis

The leakage inductances of the \(d\) - and \(q\)-windings, \(L_{\epsilon_{ds}}\) and \(L_{\epsilon_{qs}}\), are equal, and are often called \(L_{\epsilon}\), but in Simpow, they are called \(L_{\epsilon}\), why henceforth this will also be the case in this report, i.e.

\[ L_{\epsilon} = L_{\epsilon_{ds}} = L_{\epsilon_{qs}} \]  

(3.43)
In the case when the $d$-axis rotor currents are zero, $I_f = I_{kd} = 0$, the flux linkage that will be mutually coupled between the $d$-axis circuits is:

$$\psi_d + L_d I_d = -(L_d - L_0) I_d$$  \hspace{1cm} (3.44)$$

In this case, the flux linkages in the field and $d$-damper windings are:

$$\psi_f = -L_{df} I_d$$

$$\psi_{kd} = -L_{dkd} I_d$$  \hspace{1cm} (3.45)$$

When SI units are concerned, there is equal mutual flux, i.e. in this case:

$$\psi_d + L_d I_d = \psi_f = \psi_{kd}$$  \hspace{1cm} (3.46)$$

and this is also the case with some per unit systems, but not all [12]. As a consequence of the equal mutual flux, the mutual inductances are also equal. Usually this quantity is called $L_{ad}$:

$$L_{ad} = (L_d - L_0) = L_{df} = L_{dkd}$$  \hspace{1cm} (3.47)$$

In the same way, it is proved that:

$$L_{af} = (L_f - L_{af}) = (L_{kd} - L_{ckd}) = L_{fkd}$$  \hspace{1cm} (3.48)$$

Now the final reciprocal flux linkage and voltage equations, expressed in SI units, are described as:

$$\begin{bmatrix}
\psi_d \\
\psi_q \\
\psi_0 \\
\psi_f \\
\psi_{kd} \\
\psi_{mq}
\end{bmatrix} = 
\begin{bmatrix}
-L_d & 0 & 0 & L_{ad} & L_{ad} & 0 \\
0 & -L_q & 0 & 0 & 0 & L_{aq} \\
0 & 0 & -L_0 & 0 & 0 & 0 \\
-L_{ad} & 0 & 0 & L_f & L_{ad} & 0 \\
0 & -L_{aq} & 0 & 0 & 0 & L_{mk} \\
0 & 0 & 0 & 0 & 0 & L_{mq}
\end{bmatrix}
\begin{bmatrix}
I_d \\
I_q \\
I_0 \\
I_f \\
I_{kd} \\
I_{mq}
\end{bmatrix}$$  \hspace{1cm} (3.49)$$

Where

$$L_d$$ is the self-inductance of the fictive $d$-winding.

$$L_q$$ is the self-inductance of the fictive $q$-winding.

$$L_0$$ is the self-inductance of the fictive $0$-winding.

$$L_f$$ is the self-inductance of the field winding.

$$L_{kd}$$ is the self- and mutual inductances of the $k$ damper windings of the $d$-axis.

$$L_{mq}$$ is the self- and mutual inductances of the $m$ damper windings of the $d$-axis.

$$L_{ad}$$ is the mutual inductance between the rotor and the stator circuits in the $d$-axis.
$L_{dq}$ is the mutual inductance between the rotor and the stator circuits in the $q$-axis.

If redefining the matrixes used earlier, with the new reciprocal quantities, the equations can be described in a shorter manner, as:

\[
\begin{bmatrix}
\Psi_{dq0} \\
\Psi_{R}
\end{bmatrix} = 
\begin{bmatrix}
-L_{dq0dq0} & L_{dq0R} \\
L_{dq0R}^T & L_{RR}
\end{bmatrix}
\begin{bmatrix}
I_{dq0} \\
I_{R}
\end{bmatrix}
\]

\[
\begin{bmatrix}
U_{dq0} \\
U_{R}
\end{bmatrix} = 
\begin{bmatrix}
R_d & 0 \\
0 & -R_R
\end{bmatrix}
\begin{bmatrix}
I_{dq0} \\
I_{R}
\end{bmatrix} + \frac{d}{dt}\begin{bmatrix}
\Psi_{dq0} \\
\Psi_{R}
\end{bmatrix} + \begin{bmatrix}
-P_d \\
0
\end{bmatrix}\omega
\]

The changes from the original equations, do not affect the torque equation more than the change from stator quantities to $dq0$ quantities [10], which is expressed as:

\[
J \frac{d}{dt}\omega_{mech} = M_m - M_e
\]

with

\[
M_e = \frac{3}{2} \frac{\omega_{mech}}{\omega} (\Psi_d I_q - \Psi_q I_d)
\]
4 Per unit representation

When dealing with power systems, quantities are often presented in per unit values. For synchronous machines, different per unit systems are used, therefore it is important to clearly describe how the per unit system used is defined.

4.1 Machine per unit bases

The following per unit bases are used in the machine per unit system:

\[
U_{\text{base}} = \sqrt{\frac{2}{3}} U_n
\]  
\[ I_{\text{base}} = \sqrt{\frac{2}{3}} \frac{S_n}{U_n} \]  
\[
\omega_{\text{base}} = \frac{\omega_n}{2\pi f_n} = \frac{2\pi f_n}{\omega_n}
\]  
\[
\psi_{\text{base}} = \frac{U_{\text{base}}}{\omega_{\text{base}}}
\]  
\[ S_{\text{base}} = S_n = \frac{1}{2} U_{\text{base}} I_{\text{base}} \]  
\[
Z_{\text{base}} = \frac{U_{\text{base}}}{I_{\text{base}}}
\]  
\[
L_{\text{base}} = \frac{\psi_{\text{base}}}{I_{\text{base}}}
\]  
\[
\omega_{\text{mechbase}} = \frac{\omega_{\text{mech}}}{\omega}
\]  
\[
M_{\text{base}} = \frac{S_n}{\omega_{\text{mechbase}}} = \frac{3}{2} \frac{U_{\text{base}} I_{\text{base}}}{\omega_{\text{mech}}} = \frac{3}{2} \frac{\psi_{\text{base}} I_{\text{base}}}{\omega_{\text{mech}}}
\]  
\[
H = \frac{1}{2} \frac{J \omega_{\text{mechbase}}^2}{S_n}
\]

Where

- \( U_n \) is the nominal phase-phase RMS voltage of the machine.
- \( S_n \) is the nominal power of the machine.
- \( \omega_n \) is the system frequency in [rad/s]
- \( f_n \) is the system frequency in [Hz]
- \( \omega_{\text{mech}} \) is the mechanical frequency in [mech. rad/s]
- \( \omega \) is the electrical frequency in [rad/s]
- \( \omega_{\text{base}} \) is the system base frequency
- \( \omega_{\text{mechbase}} \) is the mechanical base frequency of the machine
- \( H \) is the per unit inertia constant.
4.2 Per unit flux linkage equations

4.2.1 General description

When three windings are magnetically coupled as in Figure 4.1, the flux linkage in each of them can be described as:

\[
\Psi_1 = L_1 I_1 + L_{12} I_2 + L_{13} I_3 \tag{4.11}
\]

\[
\Psi_2 = L_2 I_2 + L_{12} I_1 + L_{23} I_3 \tag{4.12}
\]

\[
\Psi_3 = L_3 I_3 + L_{13} I_1 + L_{23} I_2 \tag{4.13}
\]

Where

- \( \Psi_i \) is the flux linkage in winding \( i \)
- \( L_i \) is the self-inductance in winding \( i \)
- \( L_{ij} \) is the mutual inductance between winding \( i \) and \( j \)
- \( I_i \) is the current flowing through winding \( i \)

![Figure 4.1 Three magnetically coupled windings](image)

To simplify calculations, the quantities are often presented in per unit; the following per unit system is used by Bühler, [10], and Laible, [13].

\[
\begin{bmatrix}
\Psi_1 \\
\Psi_2 \\
\Psi_3 \\
\end{bmatrix}
= 
\begin{bmatrix}
L_{11} & L_{12} & L_{13} \\
L_{12} & L_{22} & L_{23} \\
L_{13} & L_{23} & L_{33} \\
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3 \\
\end{bmatrix}
= 
\begin{bmatrix}
\sigma_{11} & 1 & 1 \\
(1-\sigma_{12}) & 1 & \mu_2 \\
(1-\sigma_{13}) & \mu_3 & 1 \\
\end{bmatrix}
\begin{bmatrix}
x_1 \\
I_2 \\
x_3 \\
\end{bmatrix}
\tag{4.14}
\]

Where
\( \Psi_{in} \) is the base flux linkage of winding \( i \)

\( I_{in} \) is the base current of winding \( i \)

\( x_i \) is the per unit value of the reactance of winding \( i \)

\( \sigma_{ij} \) is the decrement factor of winding \( j \) relative winding \( i \)

\( \mu_i \) is the translation (or screening) factor of winding \( i \)

Here the upper-case letters are used when relating to quantities in SI units and lower-case when relating to per unit values.

The per unit value of the inductance of winding \( i \), \( l_i \), has the same value as the per unit value of the reactance of winding \( i \), \( x_i \), and is defined as:

\[
l_i = L_i \frac{I_{in}}{\Psi_{in}} = \omega n I_{in} = x_i
\]

Where

\( \omega n \) is the base frequency of the system

\( U_{in} \) is the base voltage of winding \( i \)

The decrement factor is defined as:

\[
\sigma_{ij} = 1 - \frac{L_{ij}^2}{L_i L_j}
\]

The translation factor is defined as:

\[
\mu_i = \frac{L_{ij} L_{jk}}{L_i L_{jk}}
\]

To get this description of the flux linkages, the different base factors has to be defined as:

\[
I_{2n} = \frac{\Psi_{1n}}{L_{12}}
\]

\[
I_{3n} = \frac{\Psi_{1n}}{L_{13}}
\]

\[
\Psi_{2n} = L_2 I_{2n}
\]

\[
\Psi_{3n} = L_3 I_{3n}
\]

**4.2.2 dq0-axes flux linkages**

The flux linkages of the \( dq0 \)-axes was reciprocally described using SI units in chapter 3.4 as:

\[
\begin{bmatrix}
\Psi_d \\
\Psi_q \\
\Psi_f \\
\Psi_{kd} \\
\Psi_{mq}
\end{bmatrix} =
\begin{bmatrix}
-L_{ad} & 0 & 0 & L_{ad} & 0 & I_d \\
0 & -L_{q} & 0 & 0 & 0 & I_q \\
0 & 0 & -L_{q} & 0 & 0 & I_q \\
-L_{ad} & 0 & 0 & L_{ad} & 0 & I_f \\
0 & -L_{aq} & 0 & 0 & 0 & L_{mq} \\
\end{bmatrix}
\]

The magnetic coupling is dividing the flux linkage matrix into \( d \)-, \( q \)- and \( 0 \)-axis parts. A machine with one \( d \)-axis winding and two \( q \)-axis windings, the \( d \)-, \( q \)- and \( 0 \)- axis matrixes can be written as:
Detailed Description of Synchronous Machine Models Used in Simpow

4.2.2.1 d-axis

The per unit value of the reactance, $X_{d}$, is the same as the per unit bases for the machine, $S_{base}$.

$$\left[ \begin{array}{c} \Psi_{d} \\ \Psi_{f} \\ \Psi_{ld} \end{array} \right] = \left[ \begin{array}{c} -L_{d} I_{d} \\ -L_{ad} I_{f} \\ -L_{ld} I_{ld} \end{array} \right] \left[ \begin{array}{c} L_{ad} \\ L_{f} \\ L_{ld} \end{array} \right] \left[ \begin{array}{c} I_{d} \\ I_{f} \\ I_{ld} \end{array} \right]$$

(4.23)

$$\left[ \begin{array}{c} \Psi_{q} \\ \Psi_{1q} \\ \Psi_{2q} \end{array} \right] = \left[ \begin{array}{c} -L_{q} I_{q} \\ -L_{aq} I_{q} \\ -L_{2q} I_{q} \end{array} \right] \left[ \begin{array}{c} L_{aq} \\ L_{q} \\ L_{2q} \end{array} \right] \left[ \begin{array}{c} I_{q} \\ I_{aq} \\ I_{2q} \end{array} \right]$$

(4.24)

$$\left[ \Psi_{0} \right] = \left[ -L_{0} I_{0} \right]$$

(4.25)

Comparing this with the general per unit description,

$$\left[ \begin{array}{c} \Psi_{1} \\ \Psi_{2} \\ \Psi_{3} \end{array} \right] = \left[ \begin{array}{ccc} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{array} \right] \left[ \begin{array}{c} i_{1} \\ i_{2} \\ i_{3} \end{array} \right] = \left[ \begin{array}{ccc} x_{1} & 1 & 1 \\ (1-\sigma_{12})x_{1} & 1 & \mu_{2} \\ (1-\sigma_{13})x_{1} & \mu_{3} & 1 \end{array} \right] \left[ \begin{array}{c} i_{1} \\ i_{2} \\ i_{3} \end{array} \right]$$

(4.26)

the following analysis is made for the d-, q- and 0-axis flux linkages.

4.2.2.1.1 d-axis

If $d$, $f$ and $ld$ replace the indices $1$, $2$ and $3$ respectively, and considering the definitions used, the flux linkages of the d-axis can be described as:

$$\left[ \begin{array}{c} \Psi_{d} \\ \Psi_{f} \\ \Psi_{ld} \end{array} \right] = \left[ \begin{array}{ccc} -L_{d} I_{dbase} \\ -L_{df} I_{fbase} \\ -L_{ld} I_{ld} \end{array} \right] \left[ \begin{array}{ccc} L_{df} \Psi_{dbase} \\ L_{fd} \Psi_{fbase} \\ L_{ld} \Psi_{ld} \end{array} \right] \left[ \begin{array}{c} I_{d} \\ I_{f} \\ I_{ld} \end{array} \right]$$

(4.27)

Where

$\Psi_{dbase}$ is the base flux linkage of winding $i$

$I_{dbase}$ is the base current of winding $i$

$x_{d}$ is the per unit value of the reactance of the d-winding

$\sigma_{d}$ is the decrement factor of winding $j$ relative winding $i$

$\mu_{i}$ is the translation (or screening) factor of winding $i$

The per unit base current and flux linkage of the $d$-winding, $I_{dbase}$ and $\Psi_{dbase}$, are the same as the per unit bases for the machine, $I_{base}$ and $\Psi_{base}$, i.e.

$$I_{dbase} = I_{base}$$

(4.28)

$$\Psi_{dbase} = \Psi_{base}$$

(4.29)

The per unit value of the $d$-winding inductance, $l_{d}$, has the same value as the per unit value of the reactance, $x_{d}$, and is defined as:

$$l_{d} = L_{d} \frac{I_{d}}{\Psi_{dbase}} = \omega_{s} L_{d} \frac{I_{base}}{U_{base}} = L_{d} \frac{1}{S_{base}} = x_{d}$$

(4.30)

Where

$\omega_{s}$ is the base frequency of the system.
\( U_{\text{base}} \) is the base voltage of the machine.

\( L_{\text{base}} \) is the base inductance of the machine.

The decrement factors and translation factors are defined as:

\[
\sigma_{df} = 1 - \frac{L_{df}}{L_d L_f} \quad (4.31)
\]

\[
\sigma_{dl} = 1 - \frac{L_{dl}}{L_d L_{l_d}} \quad (4.32)
\]

\[
\mu_f = \frac{L_{fi_d} L_{df}}{L_f L_{d} L_{l_d}} \quad (4.33)
\]

\[
\mu_{df} = \frac{L_{f} L_{l_d}}{L_d L_{df}} \quad (4.34)
\]

Using the definition of equal mutual inductances, \( L_{df} = L_{dl} = L_{f l_d} = L_{ad} \), and dividing all inductances with the base inductance of the machine, \( L_{\text{base}} \), all inductances can be described as the corresponding per unit reactance value.

\[
\frac{1}{L_d} x_{d} = x_d \quad (4.35)
\]

\[
\frac{1}{L_{ad}} x_{ad} = x_f \quad (4.36)
\]

\[
\frac{1}{L_{l_d}} x_{l_d} = x_{l_d} \quad (4.37)
\]

This makes the decrement factors and the translation factors somewhat unnecessary, and the flux linkages can be described as:

\[
\begin{bmatrix}
\psi_d \\
\psi_f \\
\psi_{l_d}
\end{bmatrix} =
\begin{bmatrix}
-x_d & 1 & 0 \\
-x_{ad} & 1 & x_{ad} \\
x_f & x_{ad} & 1
\end{bmatrix}
\begin{bmatrix}
i_d \\
i_f \\
i_{l_d}
\end{bmatrix}
\]

To get this description of the flux linkages, the different base factors has to be defined as:

\[
I_{\text{base}} = \frac{\Psi_{\text{base}}}{L_{df}} = \frac{\Psi_{\text{base}}}{L_{ad}} = \frac{\Psi_{\text{base}}}{L_{l_d}} x_{ad} = \frac{i_{\text{base}}}{x_{ad}} \quad (4.38)
\]

\[
\psi_{f \text{base}} = L_f I_{\text{base}} = \frac{x_f}{x_{ad}} \psi_{\text{base}} \quad (4.39)
\]

\[
\psi_{l_d \text{base}} = L_{l_d} I_{l_d \text{base}} = \frac{x_{l_d}}{x_{ad}} \psi_{\text{base}} \quad (4.40)
\]
4.2.2.2 \( q \)-axis

In the same way as for the \( d \)-axis, the flux linkages of the \( q \)-axis can be described as:

\[
\begin{bmatrix}
\psi_q \\
\psi_{1q} \\
\psi_{2q}
\end{bmatrix} =
\begin{bmatrix}
-L_1 \frac{I_{\text{base}}}{U_{\text{base}}} & L_{1q} \frac{I_{\text{base}}}{U_{\text{base}}} & L_{2q} \frac{I_{\text{base}}}{U_{\text{base}}}
-L_1 \frac{I_{\text{base}}}{U_{\text{base}}} & L_{1q} \frac{I_{\text{base}}}{U_{\text{base}}} & L_{2q} \frac{I_{\text{base}}}{U_{\text{base}}}
-L_2 \frac{I_{\text{base}}}{U_{\text{base}}} & L_{2q} \frac{I_{\text{base}}}{U_{\text{base}}} & L_{2q} \frac{I_{\text{base}}}{U_{\text{base}}}
\end{bmatrix}
\begin{bmatrix}
i_q \\
i_{1q} \\
i_{2q}
\end{bmatrix}
\]

\[
\Rightarrow
\begin{bmatrix}
\psi_q \\
\psi_{1q} \\
\psi_{2q}
\end{bmatrix} =
\begin{bmatrix}
-L_1 \frac{I_{\text{base}}}{U_{\text{base}}} & L_{1q} \frac{I_{\text{base}}}{U_{\text{base}}} & L_{2q} \frac{I_{\text{base}}}{U_{\text{base}}}
-L_1 \frac{I_{\text{base}}}{U_{\text{base}}} & L_{1q} \frac{I_{\text{base}}}{U_{\text{base}}} & L_{2q} \frac{I_{\text{base}}}{U_{\text{base}}}
-L_2 \frac{I_{\text{base}}}{U_{\text{base}}} & L_{2q} \frac{I_{\text{base}}}{U_{\text{base}}} & L_{2q} \frac{I_{\text{base}}}{U_{\text{base}}}
\end{bmatrix}
\begin{bmatrix}
i_q \\
i_{1q} \\
i_{2q}
\end{bmatrix}
\]

\[
(4.41)
\]

where

\[
i_q = L_q \frac{I_{\text{base}}}{U_{\text{base}}} = \omega x_{\text{aq}} \]

\[
(4.42)
\]

\[
L_{1q} \frac{1}{U_{\text{base}}} = \omega x_{\text{aq}}
\]

\[
(4.43)
\]

\[
L_{2q} \frac{1}{U_{\text{base}}} = \omega x_{\text{2q}}
\]

\[
(4.44)
\]

\[
I_{1q_{\text{base}} =} \frac{\psi_{\text{base}}}{L_{1q}} = \frac{\psi_{\text{base}}}{L_{1q}} = \frac{I_{\text{base}}}{x_{\text{aq}}}
\]

\[
(4.45)
\]

\[
I_{2q_{\text{base}} =} \frac{\psi_{\text{base}}}{L_{2q}} = \frac{\psi_{\text{base}}}{L_{2q}} = \frac{I_{\text{base}}}{x_{\text{2q}}}
\]

\[
(4.46)
\]

\[
\psi_{1q_{\text{base}}} = L_{1q} I_{1q_{\text{base}}} = \frac{x_{\text{aq}}}{x_{\text{aq}}} \psi_{\text{base}}
\]

\[
(4.47)
\]

4.2.2.3 \( 0 \)-axis

In the zero axis, there is only one winding, therefore the per unit value flux linkage of the zero axis is equal to

\[
\psi_0 = x_0 i_0
\]

\[
(4.48)
\]

where

\[
i_0 = L_0 \frac{I_{\text{base}}}{U_{\text{base}}} = L_0 \frac{1}{U_{\text{base}}} = x_0
\]

\[
(4.49)
\]

4.2.3 Conclusions

Rearranging the equations into the stator \( dq0 \) quantities and the rotor \( f, kd, mq \) quantities, the flux linkages matrix can be formulated, see Equation (4.38).

For a model with fewer windings in the \( d \)- and/or \( q \)-axes, the flux linkages can be described using Equation (4.38), if the rows and columns of the matrix, that are corresponding to the windings that are not included, are deleted.
Here

\( x_d \) is the direct axis synchronous reactance.

\( x_a \) is the armature leakage reactance, often called \( x_l \).

\( x_{ad} = x_d - x_a \) is the mutual reactance between the \( d \)-axis windings.

\( x_f \) is the field winding reactance.

\( x_{1d} \) is the damper winding reactance of damper 1 in the \( d \)-axis.

\( x_{1q} \) is the damper winding reactance of damper 1 in the \( q \)-axis.

\( x_{2q} \) is the damper winding reactance of damper 2 in the \( q \)-axis.

\[ (4.50) \]

4.2.4 Definition of the transient and sub-transient reactances

To be able to define the field winding reactance and the different damper winding reactances, the transient- and subtransient reactances of the \( d \)- and \( q \)-axis have to be defined.

4.2.4.1 \( d \)-axis

The subtransient reactance is defined as the initial stator flux linkage per unit of stator current, with all the stator circuits shorted and previously unenergised [12], i.e. during this subtransient time, the stator flux linkages are described as:

\[ \psi_d = x_{d''} i_d \]

\[ \psi_q = x_{q''} i_q \]  

(4.51)

where

\( x_{d''} \) is the subtransient reactance of the \( d \)-axis

\( x_{q''} \) is the subtransient reactance of the \( q \)-axis

Directly after the voltage is applied on the terminals, the rotor flux linkages, \( \psi_f, \psi_{1d}, \psi_{1q} \) and \( \psi_{2q} \), are still zero, since they can not change instantly [12]. The flux linkages of the \( d \)-axis during this subtransient time, looks like:
This gives the rotor currents, $i_f$ and $i_{id}$, during the subtransient time as:

$$
i_f = \frac{x_{ad} x_{id} - x_{q}^2}{x_f x_{id} - x_{ad}^2} i_d
$$

$$
i_{id} = \frac{x_{ad} x_f - x_{id}^2}{x_f x_{id} - x_{ad}^2} i_d
$$

Inserting these equations in the expression for $\psi_d$, gives the subtransient stator flux linkage of the d-axis:

$$
\psi_d = \left( x_{id} - \frac{x_{ad} + x_f - \frac{2 x_{ad}}{x_f} x_{id}^2}{x_f x_{id} - x_{ad}^2} \right) i_d
$$

From the definition of the subtransient reactance, it is obvious that:

$$
x_q = x_{q}^* i_q
$$

where $x_{q}^*$ is the transient reactance of the q-axis

In the same way as in the subtransient case, but with $i_{id} = 0$, the transient stator flux linkage in the d-axis is calculated as:

$$
\psi_d = \left( x_{id} - \frac{x_{ad}^2}{x_f} \right) i_d
$$

From the definition of the transient reactance, it is obvious that:

$$
x_q = x_{q}^* i_q
$$

The field winding reactance is now defined as in Equation (4.59):

$$
x_f = \frac{x_{ad}^2}{x_d - x_{q}^*}
$$

Inserting this into the equation for the subtransient reactance, gives the definition of the damper winding reactance, $x_{id}$, as:
\[ x_{1d} = x_{ad} - \frac{(x_{d} - x_{a})(x_{d} - x_{a})}{x_{d} - x_{d}^{'}} \quad (4.60) \]

### 4.2.4.2 q-axis

In the same way, the reactances of the q-axis damper windings can be defined. For the subtransient period, both damper windings are active, and during the transient period, the current through the second winding has decayed to zero. Therefore, the subtransient and the transient reactances of the q-axis damper windings are described as:

\[ x_{q} = x_{q} - \frac{x_{1q}^2 + x_{2q}^2 - 2x_{1q}x_{2q}x_{2q}}{x_{1q}x_{2q} - x_{2q}^2} \quad (4.61) \]

\[ x_{q} = x_{q} - \frac{x_{2q}^2}{x_{1q}} \quad (4.62) \]

The reactance of the first damper winding, \( x_{1q} \), is now defined as:

\[ x_{1q} = \frac{x_{2q}^2}{x_{1q}} \quad (4.63) \]

Inserting this into the subtransient reactance equation, gives the definition of the second damper winding reactance, \( x_{2q} \) as:

\[ x_{2q} = x_{pq} - \frac{(x_{pq} - x_{a})(x_{pq} - x_{q})}{x_{pq} - x_{pq}^{'}} \quad (4.64) \]

For a model with only one damper winding in the q-axis, the sub-transient reactance is used in Equation (4.63), instead of the transient reactance [3], [13].

### 4.3 Per unit voltage equations

#### 4.3.1 Stator voltages

In chapter 3.4, the stator voltages, in SI units, are described as:

\[ U_{dq0} = -R_{s}I_{dq0} + \frac{d}{dt}\Psi_{dq0} + \begin{bmatrix} -\Psi_{q} \\ \Psi_{d} \\ 0 \end{bmatrix} \omega \quad (4.65) \]

Using the per unit bases for the machine, the stator voltages can be described as:

\[ U_{dq0} = -R_{s}I_{dq0} + \frac{d}{dt}\Psi_{dq0} + \begin{bmatrix} -\Psi_{q} \\ \Psi_{d} \\ 0 \end{bmatrix} \frac{\omega}{\omega_{base}} \quad (4.66) \]

Dividing both sides with \( U_{base} \), gives the stator voltages in per unit, as:

\[ U_{dq0} = -r_{s}I_{dq0} + \frac{1}{\omega_{base}} \frac{d}{dt}\Psi_{dq0} + \begin{bmatrix} -\Psi_{q} \\ \Psi_{d} \\ 0 \end{bmatrix} \frac{\omega}{\omega_{base}} \quad (4.67) \]

#### 4.3.2 Rotor voltages

In chapter 3.4, the rotor voltages, in SI units, are described as:
### 4.3.2.1 Field winding

The field winding voltage can be written as:

\[ u_f U_{\text{base}} = R_f i_f I_{\text{base}} + \frac{d}{dt} \Psi_f \Psi_{f\text{base}} \]  

(4.70)

Defining \( U_{\text{base}} = R_f i_f \) as:

\[ U_{\text{base}} = R_f i_f \]  

(4.71)

Using this and the Equation (4.27), \( \Psi_{f\text{base}} = L_f I_{\text{base}} \) gives:

\[ \frac{\Psi_{f\text{base}}}{U_{\text{base}}} = \frac{\Psi_f}{R_f i_f} = \frac{L_f}{R_f} = \tau_f \]  

(4.72)

Dividing both sides of Equation (4.65) with \( U_{\text{base}} \), the per unit field voltage is described as:

\[ u_f = i_f + \tau_f \frac{d}{dt} \Psi_f \]  

(4.73)

where

\( \tau_f \) is the time constant of the field winding

### 4.3.2.2 Damper windings

The damper winding voltages can be described as:

\[ 0 = R_{kd} i_{kd} I_{kd\text{base}} + \frac{d}{dt} \Psi_{kd} \Psi_{kd\text{base}} \]  

(4.74)

Dividing the equations with \( R_{kd} i_{kd} \) and \( R_{mq} i_{mq\text{base}} \) respectively, and using the Equations (4.28), (4.34) and (4.35) to define:

\[ \frac{\Psi_{kd\text{base}}}{R_{kd} i_{kd\text{base}}} = \frac{L_{kd}}{R_{kd}} = \tau_{kd} \]  

(4.75)

and

\[ \frac{\Psi_{mq\text{base}}}{R_{mq} i_{mq\text{base}}} = \frac{L_{mq}}{R_{mq}} = \tau_{mq} \]  

(4.76)

gives
\[
0 = i_{kd} + \tau_{kd} \frac{d}{dt} \varphi_{kd} \\
0 = i_{mq} + \tau_{mq} \frac{d}{dt} \varphi_{mq}
\]  

(4.77)

where

\( \tau_{kd} \) are the time constants for the damper windings in the \( d \)-axis

\( \tau_{mq} \) are the time constants for the damper windings in the \( q \)-axis

### 4.3.2.3 Summary of the rotor voltage equations

From the sections 4.3.2.1 and 4.3.2.2, the per unit rotor voltages are summarised as:

\[
\begin{bmatrix}
    u_f \\
    u_q \\
    u_0
\end{bmatrix} =
\begin{bmatrix}
    i_f \\
    i_q \\
    i_0
\end{bmatrix} +
\begin{bmatrix}
    \tau_f \\
    \tau_{kd} \\
    \tau_{mq}
\end{bmatrix}
\begin{bmatrix}
    \varphi_f \\
    \varphi_{kd} \\
    \varphi_{mq}
\end{bmatrix}
\]

or shorter

\[
u_R = i_R + \tau_R \frac{d}{dt} \varphi_R
\]

(4.78)

### 4.3.3 Conclusions

For a machine with one field winding, one damper winding in the \( d \)-axis and two damper windings in the \( q \)-axis, the per unit voltage equations derived in sections 4.3.1 and 4.3.2, are assembled as:

\[
\begin{bmatrix}
    u_f \\
    u_q \\
    u_0
\end{bmatrix} =
\begin{bmatrix}
    -r_a \\
    0 - r_a \\
    0 0
\end{bmatrix}
\begin{bmatrix}
    i_d \\
    i_q \\
    i_0
\end{bmatrix} +
\begin{bmatrix}
    1 \\
    1 \\
    1
\end{bmatrix}
\begin{bmatrix}
    \frac{1}{\omega_{base}} \\
    \frac{1}{\omega_{base}} \\
    \frac{1}{\omega_{base}}
\end{bmatrix}
\begin{bmatrix}
    \frac{d}{dt} \\
    \tau_f \\
    \tau_{kd}
\end{bmatrix}
\begin{bmatrix}
    \varphi_f \\
    \varphi_{kd} \\
    \varphi_{mq}
\end{bmatrix}
\]

\[
\begin{bmatrix}
    \varphi_f \\
    \varphi_{kd} \\
    \varphi_{mq}
\end{bmatrix} =
\begin{bmatrix}
    \psi_d \\
    \psi_q \\
    \psi_0
\end{bmatrix} +
\begin{bmatrix}
    0 \\
    0 \\
    0
\end{bmatrix}
\begin{bmatrix}
    \omega \\
    \omega_{base}
\end{bmatrix}
\]

(4.80)

Where

\( r_a \) is the armature resistance.

\( \tau_f \) is the time constant of the field winding.

\( \tau_{kd} \) is the time constant of the first \( d \)-damper winding.

\( \tau_{iq} \) is the time constant of the 1\(^{st}\) \( q \)-damper winding.

\( \tau_{2q} \) is the time constant of the 2\(^{nd}\) \( q \)-damper winding.

### 4.3.4 Definition of the time constants

To be able to define time constants of the windings in the \( d \)- and \( q \)-axis, the transient- and open circuit time constants of the \( d \)- and \( q \)-axis are introduced.

In Simpow, the following definitions are used for the time constants of the windings in the \( d \)- and \( q \)-axis.

#### 4.3.4.1 \( d \)-axis

In Laible, [13], the time constants of the \( d \)-axis are defined as:
\[ \tau_f = \tau_{d0} - \tau_{1d} \]  
\[ \tau_{1d} = \frac{\tau_{d0}}{\sigma_{f1d}} = \frac{\tau_{d0}'}{1 - \frac{x_{ad}}{x_{f1d}}} \]  
(4.81)  
(4.82)

where

\( \tau_{d0} \) is the transient open circuit time constant of the \( d \)-axis.

\( \tau_{d0}' \) is the subtransient open circuit time constant of the \( d \)-axis.

\( \sigma_{f1d} \) is the decrement factor of winding \( ld \) relative winding \( f \).

This definition is approximate, and is only valid when \( \tau_f \gg \tau_{1d} \), i.e. when \( \frac{L_f}{R_f} \gg \frac{L_{1d}}{R_{1d}} \), which mostly is the case [13].

In the case with no damper winding is modelled in the \( d \)-axis, \( \tau_f = \tau_{d0}' \).

### 4.3.4.2 \( q \)-axis

Analogously, in the \( q \)-axis, the time constants are defined as:

\[ \tau_{iq} = \tau_{q0} - \tau_{2q} \]  
\[ \tau_{2q} = \frac{\tau_{q0}'}{\sigma_{iq2q}} = \frac{\tau_{q0}'}{1 - \frac{x_{aq}}{x_{1q}x_{2q}}} \]  
(4.83)  
(4.84)

where

\( \tau_{q0} \) is the transient open circuit time constant of the \( q \)-axis.

\( \tau_{q0}' \) is the subtransient open circuit time constant of the \( q \)-axis.

\( \sigma_{iq2q} \) is the decrement factor of winding \( 2q \) relative winding \( 1q \)

This definition is approximate, and is only valid when \( \tau_{iq} \gg \tau_{2q} \), i.e. when \( \frac{L_{iq}}{R_{iq}} \gg \frac{L_{2q}}{R_{2q}} \).

In the case with only one damper winding modelled in the \( q \)-axis, \( \tau_{iq} = \tau_{q0}' \).

### 4.4 Per unit torque equations

In chapter 3.4, the torque is described, in SI units, as:

\[ J \frac{d}{dt} \omega_{\text{mech}} = M_m - M_e \]  
(4.85)

with

\[ M_e = \frac{3}{2} \frac{\omega_{\text{mech}}}{\omega} (\Psi_d l_q - \Psi_q l_q) \]  
(4.86)

Using the same per unit bases for the machine as the previous sub-chapter, with the additional bases:
\[ \omega_{\text{mech base}} = \frac{\omega_{\text{mech}}}{\omega_n} \]
\[ M_{\text{base}} = \frac{S_n}{\omega_{\text{mech base}}} = \frac{3}{2} \frac{\psi_{\text{base}}}{\omega_{\text{mech}}} \]
\[ H = \frac{1}{2} J \omega_{\text{mech base}}^2 \]

Where \( H \) is the per unit inertia constant. The torque can be written as:
\[ \frac{2H S_n}{\omega_{\text{mech base}}} \frac{d}{dt} \omega_{\text{mech base}} = (m_m - m_e) M_{\text{base}} \] (4.88)

Dividing both sides with \( M_{\text{base}} \), gives the torque in per unit, as:
\[ 2H \frac{d}{dt} \frac{\omega}{\omega_n} = m_m - m_e \] (4.89)

In the above equation,
\[ \Delta \omega \text{ is the rotor speed deviation} \]
\[ \theta \text{ is the deviation angle of the rotor} \]

The per unit electrical torque, \( m_e \), is now:
\[ m_e = \psi_{d} i_q - \psi_{q} i_d \] (4.91)

### 4.4.1 Fundamental frequency models

For the fundamental frequency models, an extra term must be added to the electrical torque, the negative-sequence breaking torque, \( m_{e2} \), [9]. That is, for the transient stability simulations, the per unit electrical torque is changed to:
\[ m_e = (\psi_{d} i_q - \psi_{q} i_d) + m_{e2} = (\psi_{d} i_q - \psi_{q} i_d) + (r_2 - r_a) i_2^2 \] (4.92)

where
- \( r_2 \) is the negative sequence resistance.
- \( r_a \) is the armature resistance.
- \( i_2 \) is the negative sequence current.

### 4.4.2 Models without damper windings

The damping torque is linearly dependent of the speed deviations of the machine [12], and is the result of an electrical effect. I.e. if an excess torque changes the speed of the machine, this relative movement generates currents in the rotor circuits. Losses caused by these currents will strive to counteract the change in motion [15].

When no damper windings are modelled, no currents will occur in the rotor circuits, i.e. no electrical damping torque will be included in the model.

To obtain damping in the simpler models without rotor damper circuits, a substitute for the electrical damping torque must be implemented in the torque equation [9]. This damping torque is called \( m_d \), and is defined as:
Detailed Description of Synchronous Machine Models Used in Simpow

\[ m_d = D \frac{\Delta \omega}{\omega_n} \text{[p.u.]} \quad (4.93) \]

where \( D \) is the damping factor.

In these cases, the per unit torque equation is described as:

\[ 2H \frac{d \omega}{dt} \frac{\omega}{\omega_n} = m_m - m_e - m_d \quad (4.94) \]

### 4.5 Global - local per unit transformation

At the node where the machine is connected to the network, the quantities are described using system per unit bases. A per unit transformation constant is therefore needed when performing a global-local transformation in per unit.

In Simpow, the global voltages are transformed to the local system, and the local currents are transformed to the global system, why the transformation constants \( C_U \) and \( C_I \) are defined as:

\[
C_U = \frac{U_{\text{node base}}}{U_n} \\
C_I = \frac{S_{\text{node base}}}{U_n S_{\text{net base}}} 
\]

where

- \( U_{\text{node base}} \) is the nominal phase-phase voltage for the machine node
- \( S_{\text{net base}} \) is the 3-phase network power base

The transformation constants are used in the transformation equations in the following manner:

for the global to local transformation:

\[
U_d = [U_\alpha \sin \delta - U_\beta \cos \delta] C_U = [U_\alpha \cos \theta + U_\beta \sin \theta] C_U 
\]

\[
U_q = [U_\alpha \cos \delta + U_\beta \sin \delta] C_U = [-U_\alpha \sin \theta + U_\beta \cos \theta] C_U 
\]

and the local to global transformation looks like:

\[
I_\alpha = [I_d \sin \delta + I_q \cos \delta] C_I = [I_d \cos \theta - I_q \sin \theta] C_I 
\]

\[
I_\beta = [-I_d \cos \delta + I_q \sin \delta] C_I = [I_d \sin \theta + I_q \cos \theta] C_I 
\]

Where

- \( \theta \) is the electrical displacement angle for an instantaneous value model.
- \( \delta \) is the electrical displacement angle for a fundamental frequency model.

For the fundamental frequency models, transforms are made between the global \( dq \)-system and the local \( dq0 \)-system. For the instantaneous value models, transforms are made between the global positive-, negative- and 0-sequence systems and the local \( dq0 \)-system.
5 Modelling

Within Simpow, there are four different synchronous machine models in the standard library; a list of them is shown below.

Type 1. Model with four rotor windings including saturation; one field winding, two d-axis and one q-axis damper windings
Type 1A. As Type 1, but without saturation
Type 2. Model with three rotor windings including saturation; one field winding, one d-axis and one q-axis damper windings
Type 2A. As Type 2, but without saturation
Type 3. Model with one rotor windings including saturation; 1 field winding
Type 3A. As Type 3, but without saturation
Type 4. Model with constant voltage behind transient reactance

In this chapter, first the differences between the fundamental frequency, and instantaneous value models are discussed, then the initial conditions for the machine models are described. At last, the four different synchronous machine models are discussed, starting the simplest model, Type 4, and continuing with the more and more advanced models, Type 3A, 2A and 1A.

5.1 Transient-/Machine stability

When the machine stability of a power system is being simulated, instantaneous value models of the power system components are used. In Simpow, this simulation mode is called Masta. For transient stability simulations, fundamental frequency models are used; this mode is in Simpow called Transta.

For the fundamental frequency models, the following changes are made:

- The transformer voltages, \( \frac{d}{dt} \psi_d \) and \( \frac{d}{dt} \psi_q \), are neglected.
- The models are only used for the positive sequence quantities, i.e. the zero axis voltage and flux linkage equations, \( u_0 = -r_a \dot{i}_0 + \frac{1}{\omega_0} \frac{d}{dt} \psi_0 \) and \( \psi_0 = -x_0 \dot{i}_0 \), are neglected.
- The negative and zero sequences are modelled as two impedance circuits, with the resistance and reactance: \( r_2, x_2 \) and \( r_0, x_0 \) respectively.
- The electrical torque is modelled with an extra term, the negative-sequence breaking torque: \( m_{e2} = (r_2 - r_a) i_2^2 \).

With the transformer voltages neglected, the stator voltage equations only contains fundamental frequency components, and appear as algebraic equations. When also the zero axis voltage equation is neglected, the fundamental frequency models will have three state variables less than the instantaneous value models. The machine models discussed in the following sub-chapters are of order 5, 6, 8 and 9, referred to the instantaneous value order; i.e. the fundamental frequency models are of order 2, 3, 5 and 6. In Simpow, these models are named the synchronous machine models Type 4, Type 3, Type 2 and Type 1 respectively.
5.2 Initial conditions at steady state

At time zero, the initial conditions are calculated. From the load-flow calculation, the initial node voltage and angle, \( u_0 \) and \( \phi_0 \), and the active- reactive power out from the generator node, \( p_{th_0}, q_{th_0} \), are known.

At steady state, all time derivatives are zero. The machine is assumed to run under symmetrical conditions, \( \psi_0 = u_0 = i_0 = \phi_0 = 0 \). When deriving the stator circuit quantities, \( u_{d0}, u_{q0}, \psi_{d0}, \psi_{q0}, i_{d0} \) and \( i_{q0} \) saliency is neglected, i.e. \( x_d = x_q \), but when deriving the rotor circuit quantities, \( \psi_{f0}, \psi_{df0}, \psi_{qf0}, \psi_{qf0} \) and \( i_{f0} \), saliency is included. With these considerations in mind, the voltage equations derived in the previous chapters, see Equation (4.80), will be changed to:

\[
\begin{bmatrix}
  u_{d0} \\
  u_{q0} \\
  u_{f0} \\
  \psi_{d0} \\
  \psi_{q0} \\
  \psi_{f0} \\
  \psi_{d0} \\
  \psi_{q0}
\end{bmatrix} =
\begin{bmatrix}
  r_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  -r_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & -r_a & 0 & 0 & 0 & 0 & 0 & 0 \\
  x_{ad} & 0 & 0 & x_f & 0 & 0 & 0 & 0 \\
  x_{qd} & 0 & 0 & 0 & x_{ad} & 0 & 0 & 0 \\
  x_{ad} & 0 & 0 & 0 & 0 & x_{ad} & 0 & 0 \\
  x_{qd} & 0 & 0 & 0 & 0 & 0 & x_{ad} & 0 \\
  x_{qd} & 0 & 0 & 0 & 0 & 0 & 0 & x_{ad}
\end{bmatrix}
\begin{bmatrix}
  i_{d0} \\
  i_{q0} \\
  i_{f0} \\
  \psi_{d0} \\
  \psi_{q0} \\
  \psi_{f0} \\
  \psi_{d0} \\
  \psi_{q0}
\end{bmatrix} +
\begin{bmatrix}
  0 \\
  0 \\
  0 \\
  \omega \\
  \omega \\
  \omega \\
  \omega \\
  \omega
\end{bmatrix}
\begin{bmatrix}
  \psi_{d0} \\
  \psi_{q0}
\end{bmatrix}
\]  
(5.1)

Where the extra index, \( \theta \), is referring to the initial moment of time.

Obviously, the currents in the damper windings are zero, i.e. \( i_{1d0} = i_{1d0} = i_{1d0} \). This was expected, since the damper windings only affect the machine during unstable conditions. This changes the flux linkage equation (4.46) to:

\[
\begin{bmatrix}
  \psi_{d0} \\
  \psi_{q0} \\
  \psi_{f0} \\
  \psi_{d0} \\
  \psi_{q0} \\
  \psi_{f0} \\
  \psi_{d0} \\
  \psi_{q0}
\end{bmatrix} =
\begin{bmatrix}
  -x_d & 0 & 0 & x_d & 0 \\
  0 & -x_d & 0 & 0 & 0 \\
  -x_{ad} & 0 & 1 & 0 & 0 \\
  x_f & 0 & 0 & x_{ad} & 0 \\
  x_{qf} & 0 & 0 & 0 & x_{ad} \\
  x_{nf} & 0 & 0 & 0 & 0 \\
  0 & -x_{qf} & 0 & x_{ad} & 0 \\
  0 & -x_{qf} & 0 & 0 & x_{ad}
\end{bmatrix}
\begin{bmatrix}
  i_{d0} \\
  i_{q0} \\
  i_{f0} \\
  \psi_{d0} \\
  \psi_{q0} \\
  \psi_{f0} \\
  \psi_{d0} \\
  \psi_{q0}
\end{bmatrix}
\]  
(5.2)

The initial node voltage is defined as

\[
u_{th} = u_{d0} + j u_{q0}
\]  
(5.3)

Inserting the voltages from Equation (5.1), gives:

\[
u_{th} = -r_a i_{d0} + \psi_{q0} - \frac{\omega}{\omega_n} \psi_{q0} + j(-r_a i_{q0} - \frac{\omega}{\omega_n} \psi_{d0})
\]  
(5.4)

Now, the machine is assumed to run at synchronous speed, i.e. \( \omega = \omega_n \), and with the flux linkages from Equation (5.2), \( u_{th} \) can be written as:

\[
u_{th} = -r_a i_{d0} + \psi_{q0} - \frac{\omega}{\omega_n} \psi_{q0} + j(-r_a i_{q0} - \frac{\omega}{\omega_n} \psi_{d0})
\]  
(5.5)

Rearranging, and introducing the initial steady-state current out from the generator to the node, \( i_{th} = i_{d0} + j i_{q0} \) and from Equation (5.1), \( u_{th} = i_{f0} \). This gives the final equation for the node voltage as:

\[
u_{th} = -i_{th}(r_a + j x_d) - j u_{f0}
\]  
(5.6)

where
$u_{f0}$ is the initial steady-state field voltage

The initial steady-state model can thus be viewed as the initial field voltage, $u_{f0}$, behind an impedance, $r_a + jx_d$, see Figure 5.1.

From the steady-state equivalent circuit, the internal load angle, $\delta_i$, and the $dq$-voltages, $u_d$ and $u_q$, can be calculated [9]. Using the steady-state equivalent circuit, a voltage diagram is drawn, see Figure 5.2, and the Equations (5.7-5.11) are stated.

$$u_{d0} = u_{f0} \sin \delta_i$$  \hspace{1cm} (5.7)

$$u_{q0} = u_{f0} \cos \delta_i$$  \hspace{1cm} (5.8)

Where $\sin \delta_i$ and $\cos \delta_i$ can be described as:

$$\sin \delta_i = \frac{x_d i_{10} \cos \phi_i - r_d i_{10} \sin \phi_i}{u_{f0}}$$  \hspace{1cm} (5.9)

$$\cos \delta_i = \frac{u_{f0} + r_d i_{10} \cos \phi_i + x_d i_{10} \sin \phi_i}{u_{f0}}$$  \hspace{1cm} (5.10)

The initial field winding voltage is derived using the Pythagorean theorem.

$$u_{f0} = \sqrt{\left(u_{i0} + r_d i_{10} \cos \phi_i + x_d i_{10} \sin \phi_i\right)^2 + \left(x_d i_{10} \cos \phi_i - r_d i_{10} \sin \phi_i\right)^2}$$  \hspace{1cm} (5.11)

With the initial power, out from the generator into the node, defined as:
\[
p_{r0} + jq_{r0} = u_{r0}i_{r0}^* = (u_{d0} + ju_{q0})(i_{d0} - ji_{q0}) \tag{5.12}
\]

gives the real and imaginary power as:

\[
p_{r0} = u_{r0}i_{r0} \cos \varphi_i \tag{5.13}
\]

\[
q_{r0} = u_{r0}i_{r0} \sin \varphi_i \tag{5.14}
\]

The initial field voltage, \( u_{f0} \), the \( dq \)-voltages, \( u_{d0}, u_{q0} \), and the internal load angle, \( \delta_i \), can be written as:

\[
u_{f0} = u_{r0} \sqrt{1 + \left( \frac{p_{r0}}{u_{r0}} \right)^2 + \left( \frac{q_{r0}}{u_{r0}} \right)^2} \left( r_{d}^2 + x_{d}^2 \right) + 2 \left( r_{a} \frac{p_{r0}}{u_{r0}} + x_{a} \frac{q_{r0}}{u_{r0}} \right) \tag{5.15}
\]

\[
u_{d0} = u_{r0} \frac{x_{d} p_{r0} - r_{a} q_{r0}}{u_{r0}} \tag{5.16}
\]

\[
u_{q0} = u_{r0} \frac{u_{r0} + r_{a} p_{r0} + x_{d} q_{r0}}{u_{r0}} \tag{5.17}
\]

\[
\delta_i = \arctan \left( \frac{u_{d0}}{u_{q0}} \right) \tag{5.18}
\]

Equations (5.14)-(5.17) are the initial settings of the machine, and all other variables can be calculated from these.

The initial values of the currents \( i_{d0} \) and \( i_{q0} \) can be derived from the power equation, (5.12), as:

\[
i_{d0} = u_{d0} \frac{p_{r0}}{u_{r0}} + u_{q0} \frac{q_{r0}}{u_{r0}} \tag{5.19}
\]

\[
i_{q0} = u_{q0} \frac{p_{r0}}{u_{r0}} - u_{d0} \frac{q_{r0}}{u_{r0}}
\]

From Equation (5.1), the flux linkages, \( \psi_{d0} \) and \( \psi_{q0} \), are described as:

\[
\begin{bmatrix}
\psi_{d0} \\
\psi_{q0}
\end{bmatrix} = \begin{bmatrix}
\frac{\omega_f}{\omega} & -u_{d0} \\
\frac{\omega_f}{\omega} & -u_{q0}
\end{bmatrix} \begin{bmatrix}
0 & 0 & r_a \\
0 & 0 & -r_a
\end{bmatrix} \begin{bmatrix}
i_{d0} \\
i_{q0}
\end{bmatrix}
\]

Equation (5.2) can be rearranged, with the magneto motive forces of the \( d \)- and \( q \)-axis, \( W_{d0} \) and \( W_{q0} \), separated from the first two rows:

\[
\begin{bmatrix}
\psi_{d0} \\
\psi_{q0}
\end{bmatrix} = \begin{bmatrix}
-x_a & 0 & 0 \\
0 & -x_a & 0 \\
0 & 0 & 1 - x_{df} \\
0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
i_{d0} \\
i_{q0} \\
i_{f0}
\end{bmatrix} + \begin{bmatrix}
W_{d0} \\
W_{q0}
\end{bmatrix} + \begin{bmatrix}
x_{ad} W_{d0} \\
x_{ad} W_{d0} \\
x_{ad} W_{q0} \\
x_{ad} W_{q0}
\end{bmatrix} \tag{5.20}
\]

\[
\begin{bmatrix}
\psi_{a0} \\
\psi_{a0}
\end{bmatrix} = \begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix} \begin{bmatrix}
i_{d0} \\
i_{q0} \\
i_{f0}
\end{bmatrix} + \begin{bmatrix}
x_{ad} W_{d0} \\
x_{ad} W_{d0} \\
x_{ad} W_{q0} \\
x_{ad} W_{q0}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\psi_{a0} \\
\psi_{a0}
\end{bmatrix} = \begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix} \begin{bmatrix}
i_{d0} \\
i_{q0} \\
i_{f0}
\end{bmatrix} + \begin{bmatrix}
x_{ad} W_{d0} \\
x_{ad} W_{d0} \\
x_{ad} W_{q0} \\
x_{ad} W_{q0}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\psi_{a0} \\
\psi_{a0}
\end{bmatrix} = \begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix} \begin{bmatrix}
i_{d0} \\
i_{q0} \\
i_{f0}
\end{bmatrix} + \begin{bmatrix}
x_{ad} W_{d0} \\
x_{ad} W_{d0} \\
x_{ad} W_{q0} \\
x_{ad} W_{q0}
\end{bmatrix}
\]
From row 1 and 2 in (5.20), \( W_{d0} \) and \( W_{q0} \) are extracted, so the unknown flux linkages, \( \psi_{f0}, \psi_{f1d0}, \psi_{f1q0} \) and \( \psi_{f2q0} \), can be calculated.

The magneto motive forces expressed as a function of currents, are:

\[
W_{d0} = i_{f0} - i_{d0} x_{ad} \\
W_{q0} = -i_{q0} x_{aq}
\]  

(5.21)

Without saturation, equations (5.16) and (5.17) gives the final value of \( u_{d0} \) and \( u_{q0} \), and the other variables can be calculated as in equations (5.18) – (5.21).

In the case when saturation is included, equations (5.16) and (5.17) gives start values for \( u_{d0} \) and \( u_{q0} \). The initial state can only be obtained after an iteration procedure by means of equations (5.18) – (5.22), with the change in Equation (5.20) from magneto motive forces to saturation functions, i.e. change \( W_d \) and \( W_q \) to \( f_d(W_d) \) and \( f_q(W_q) \).

\[
u_{d0}^2 = u_{d0}^2 + u_{q0}^2
\]  

(5.22)

The concept of saturation is discussed in chapter 5.7.

5.3  Type 4

In classical theory, the machine is modelled as a constant voltage behind a transient reactance; this is generally called the classical model. The Type 4 model in Simpow, is a modified classical model, and can be used for transient stability studies. However, in most cases, the more accurate models, Type 1 and 2, are to prefer, since in these models there is a more adequate representation of the electrical damping [15].

5.3.1  Assumptions for the classical model

For a typical classical model, as can be found in the literature, see section “5.3.1 Classical Model” in Kundur [9], the following assumptions are made:

1. The flux linkages of the field winding and of the first q-damper winding are constant.
2. The effect from other damper circuits is neglected.
3. Transient saliency is ignored, i.e. \( x_{d} = x_{q} \).
4. The constant voltage and reactance are not affected by speed variations.
5. Armature resistance, \( R_a \), can be neglected.

In section “4.9.3.3.1 Classical Model” IEEE Brown Book [9], the following additional assumptions are made:

6. The shaft mechanical power remains constant.
7. Damping is non-existent.

If the system is stable when these assumptions are made, stability will likely occur if any or all of the above factors are taken under consideration. However, if the system would be unstable with the assumptions made, the system may in fact be stable [9].

5.3.2  Assumptions in Type 4

In Type 4, the same assumptions are made as for the classical model, with the following remarks for the last three points in the list above

5. The armature resistance, \( R_a \), is included, but the default value is set to zero.
6. A turbine can be connected to the model, but if no turbine is connected, the shaft mechanical power is constant.

7. Damping is included in the model, by the means of the damping coefficient, $D$, but the default value is set to zero.

Beside these remarks,

- Saliency is included, if $x_q$ is given a value that differs from $x_d$.
- Both a synchronous and a transient reactance are modelled in the $d$-axis, if $x_d$ is given a value that differs from $x_d'$.

Together, these changes make the Type 4 a little different from the typical classical model.

To obtain a classical model in Simpow, as defined in the previous section, using the Type 4 model, the user should specify the quantities: $S_n$, $H$, $U_n$, $x_d'$, $x_d$ and $x_q$. With $x_d = x_q$ and $x_d' = x_d - \varepsilon$. Here $\varepsilon$ is a positive constant, due to a mathematical constraint in Simpow, so that $x_d' < x_d$, e.g. $\varepsilon = 10^{-6}$.

5.3.3 Description of Type 4

Type 4 is a model with neither a field winding nor any damper windings. Therefore, as stated in chapter 4.4.2, the only way to obtain damping in this model, is to include the substitute electrical damping torque $m_d$, i.e. to include the damping coefficient, $D$.

5.3.3.1 Instantaneous value model

With these considerations in mind, the machine equations derived in chapter 1, will for the instantaneous value Type 4 model be changed to:

$$
\begin{bmatrix}
\psi_d \\
\psi_q \\
\psi_0 \\
\psi_{f0}
\end{bmatrix} = 
\begin{bmatrix}
-x_d' & 0 & 0 & 0 \\
0 & -x_q & 0 & 0 \\
0 & 0 & -x_0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
i_d \\
i_q \\
i_0 \\
i_{f0}
\end{bmatrix} + 
\begin{bmatrix}
\psi_{f0} \\
0 \\
0 \\
-x_{ad}i_{d0}
\end{bmatrix}
$$

(5.23)

$$
\begin{bmatrix}
u_d \\
u_q \\
u_0 \\
u_{f0}
\end{bmatrix} = 
\begin{bmatrix}
-r_d & 0 & 0 & 0 \\
0 & -r_q & 0 & 0 \\
0 & 0 & -r_0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
i_d \\
i_q \\
i_0 \\
i_{f0}
\end{bmatrix} + 
\begin{bmatrix}
\frac{1}{\omega_n} \\
\frac{1}{\omega_n} \\
\frac{1}{\omega_n} \\
\tau_f
\end{bmatrix}
\begin{bmatrix}
\psi_d \\
\psi_q \\
\psi_0 \\
\psi_{f0}
\end{bmatrix} + 
\begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
$$

(5.24)

$$
2H \frac{d\Delta\omega}{dt} = m_m - m_e - m_d
$$

(5.25)

$$
\frac{d}{dt} \delta = \Delta\omega
$$

(5.26)

In the field winding voltage equation, (5.24), the term $\tau_f \frac{d}{dt} \psi_{f0}$ is equal to zero, but is included to show the resemblance with higher order models.

It is obvious to see that there are five state variables for this model, $\psi_d$, $\psi_q$, $\psi_0$, $\Delta\omega$ and $\delta$; i.e. the Type 4 instantaneous value model is a 5th order model.

With the exception that there is no d-damper winding, i.e. $x_{1d} = \infty$, the constant field winding flux linkage is calculated as in the steady-state model, see Equation (5.2).
5.3.3.1 Equivalent circuit

The $d$-, $q$- and $0$-axis equivalent circuits for the instantaneous value Type 4 model, can now be viewed as in Figure 5.3.

![Figure 5.3 Equivalent dq0-circuits for Type 4, instantaneous value model in per unit](image)

Figure 5.3 Equivalent $dq0$-circuits for Type 4, instantaneous value model in per unit

All models discussed in this chapter will have the same $0$-sequence model, i.e. the same equivalent $0$-circuit; hence, it is only viewed here.

5.3.3.2 Fundamental frequency model

Implementing in Equations (5.23)-(5.26), the changes between the instantaneous value model and the fundamental frequency model, stated in chapter 5.1, the latter model is derived as:

$$
\begin{bmatrix}
\psi_d \\
\psi_q \\
\psi_{f0}
\end{bmatrix} =
\begin{bmatrix}
-x_d' & 0 & 0 \\
0 & -x_q & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
i_d \\
i_q \\
i_{f0}
\end{bmatrix} +
\begin{bmatrix}
\psi_{f0} \\
0 \\
-\frac{x_d^2}{x_f}i_d
\end{bmatrix}
$$

(5.27)

$$
\begin{bmatrix}
u_d \\
u_q \\
u_{f0}
\end{bmatrix} =
\begin{bmatrix}
-r_d & 0 & 0 \\
0 & -r_q & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
i_d \\
i_q \\
i_{f0}
\end{bmatrix} +
\tau_f \frac{d}{dt}
\begin{bmatrix}
0 \\
0 \\
\psi_{f0}
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
-\psi_q \\
\psi_d
\end{bmatrix}
$$

(5.28)

$$
2H \frac{d\Delta \omega}{dt} = m_m - m_e - m_d
$$

(5.29)

$$
\frac{d}{dt} \delta = \Delta \omega
$$

(5.30)

with

$$
m_e = \psi_d i_d - \psi_q i_q + m_e2
$$

(5.31)

In the field winding voltage equation, (5.28), the term $\tau_f \frac{d}{dt}\psi_{f0}$ is equal to zero, but is included to show the resemblance with higher order models.

As stated earlier, this model has only two state variables: $\Delta \omega$ and $\delta$. Thus, the Type 4 fundamental frequency model is of order 2.

5.3.3.2.1 Equivalent circuit

If $x_d' = x_d = x_q$, an equivalent circuit can be drawn for the positive sequence quantities of this fundamental frequency model, see Figure 5.4. Where $u = u_d + ju_q$ and $i = i_d + ji_q$. 

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For the fundamental frequency models, the negative- and 0-sequence components are modelled as two impedance circuits with the resistance and reactance: $r_2$, $x_2$ and $r_0$, $x_0$ respectively, see Figure 5.5.

All models discussed in this chapter have the same model for the unsymmetrical components, i.e. the same equivalent negative- and 0-sequence circuits; hence, they are only viewed here.

### 5.4 Type 3A

In this chapter, the synchronous machine model Type 3A is described. This model is a simplification of Type 3, where the saturation effect is included. In chapter 5.7, the effects of the saturation are described.

To improve the classical model, a field winding can be included. In this way, the time variations of the direct axis reactance will be considered. Thus, the constant field winding flux linkage, in Type 4, is changed to be dependent of the d-axis current and of the changing field winding current, this also makes the d-axis flux linkage to be dependent of the field winding current. In addition to these changes, speed variations are included in the model.

Still, no damper windings are included in the model, i.e. the only way to obtain damping in this model, is to include the substitute electrical damping torque $m_d$.

#### 5.4.1 Instantaneous value model

Implementing this in the machine equations for the Type 4 model, the instantaneous value Type 3A model, will look like:
5.4.1.1 Equivalent circuit

To be able to draw an equivalent circuit, the flux linkage and voltage equations are

\[ \begin{bmatrix} \psi_d \\ \psi_q \\ \psi_0 \\ \psi_f \end{bmatrix} = \begin{bmatrix} -x_d & 0 & 0 & 1 \\ 0 & -x_q & 0 & 0 \\ 0 & 0 & -x_0 & 0 \\ \frac{-x_{ad}}{s_f} & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_0 \\ i_f \end{bmatrix} \] (5.32)

\[ \begin{bmatrix} u_d \\ u_q \\ u_0 \\ u_f \end{bmatrix} = \begin{bmatrix} -r_d & 0 & 0 & 0 \\ 0 & -r_q & 0 & 0 \\ 0 & 0 & -r_0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_0 \\ i_f \end{bmatrix} + \begin{bmatrix} \frac{1}{\tau_n} \\ \frac{1}{\omega_n} \end{bmatrix} \begin{bmatrix} \psi_d \\ \psi_q \end{bmatrix} + \begin{bmatrix} -\psi_0 \\ 0 \end{bmatrix} \] (5.33)

\[ 2H \frac{d\Delta \omega}{dt} = m_m - m_e - m_d \] (5.34)

\[ \frac{d}{dt} \delta = \Delta \omega \] (5.35)

There are one more state variable for this model than for the Type 4 model, \( \psi_t \), i.e. the Type 3A instantaneous value model is a 6th order model.

### 5.4.1.1 Equivalent circuit

To be able to draw an equivalent circuit, the flux linkage and voltage equations are also stated in SI-units, see Equations (5.36) and (5.37).

\[ \begin{bmatrix} \Psi_d \\ \Psi_q \\ \Psi_0 \\ \Psi_f \end{bmatrix} = \begin{bmatrix} -L_{ad} & 0 & 0 & L_{ad} \\ 0 & -L_{dq} & 0 & 0 \\ 0 & 0 & -L_0 & 0 \\ -L_{af} & 0 & 0 & L_f \end{bmatrix} \begin{bmatrix} I_d \\ I_q \\ I_0 \\ I_f \end{bmatrix} \] (5.36)

\[ \begin{bmatrix} U_d \\ U_q \\ U_0 \\ U_f \end{bmatrix} = \begin{bmatrix} -R_a & 0 & 0 & 0 \\ 0 & -R_q & 0 & 0 \\ 0 & 0 & -R_0 & 0 \\ 0 & 0 & 0 & R_f \end{bmatrix} \begin{bmatrix} I_d \\ I_q \\ I_0 \\ I_f \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \Psi_d \\ \Psi_q \\ \Psi_0 \\ \Psi_f \end{bmatrix} + \begin{bmatrix} -\Psi_0 \\ 0 \end{bmatrix} \omega \] (5.37)

Where the upper-case letters are used when relating to quantities in SI units.

As described in chapter 3.4, the mutual inductance can be described as the difference between the self-inductance and the leakage inductance. Therefore the self-inductance can be described as the sum of the mutual inductance and the leakage inductance, and the flux linkage equations can be described as:

\[ \begin{bmatrix} \Psi_d \\ \Psi_q \\ \Psi_0 \\ \Psi_f \end{bmatrix} = \begin{bmatrix} -(L_{ad} + L_a) & 0 & 0 & L_{ad} \\ 0 & -L_{dq} & 0 & 0 \\ 0 & 0 & -L_0 & 0 \\ -L_{af} & 0 & 0 & L_f \end{bmatrix} \begin{bmatrix} I_d \\ I_q \\ I_0 \\ I_f \end{bmatrix} \] (5.38)

The \( d \)-axis equivalent circuit can be described either as in Figure 5.6, or by using the leakage inductance as in Figure 5.7. This latter circuit is often easier to understand, and it is easy to declare the equations of the circuit, therefore the following models will be described this way, without showing the leakage inductance in the flux linkage equations.

The \( q \)-axis is modelled in the same way as Type 4, see Figure 5.3, with the difference that the speed variations are included. An equivalent circuit for the \( q \)-axis is viewed in Figure 5.8.
The equivalent $\theta$-circuit for Type 3A is viewed in Figure 5.3.

### 5.4.2 Fundamental frequency model

From the set of equations for the instantaneous value model, see Equations (5.32)-(5.35), and implementing the changes stated in chapter 5.1, the fundamental frequency model is derived.

\[
\begin{bmatrix}
\psi_d \\
\psi_q \\
\psi_f
\end{bmatrix} =
\begin{bmatrix}
-x_d & 0 & 1 \\
0 & -x_q & 0 \\
-x_{ad} & 0 & 1
\end{bmatrix}
\begin{bmatrix}
I_d \\
I_q \\
I_f
\end{bmatrix} = 0
\]

(5.39)

\[
\begin{bmatrix}
u_d \\
u_q \\
u_f
\end{bmatrix} =
\begin{bmatrix}
-x_d & 0 & 0 \\
0 & -x_q & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
I_d \\
I_q \\
I_f
\end{bmatrix} + \tau_f \frac{d}{dt} \begin{bmatrix}
0 \\
0 \\
\psi_f
\end{bmatrix} + \begin{bmatrix}
-\psi_q \\
0
\end{bmatrix} \frac{\omega}{\omega_o}
\]

(5.40)

\[
2H \frac{d\Delta \omega}{dt} = m_{m} - m_{e} - m_{d}
\]

(5.41)

\[
\frac{d}{dt} \delta = \Delta \omega
\]

(5.42)

with
\[ m_e = \psi_q i_d - \psi_d i_q + m_e \]  

(5.43)

In the same manner as for the instantaneous value model, it is verified that this model has one more state variable than the Type 4 model. Thus, the Type 3A fundamental frequency model is of order 3.

5.5 Type 2A

In this chapter, the synchronous machine model Type 2A is described. This model is a simplification of Type 2, where the saturation effect is included. In chapter 5.7, the effects of the saturation are described.

Further improvements are made if damper windings in the d- and q-axis are included in the model. In the case of one damper winding in each axis, the fast effects of the saturation are described. Furthermore, the effects of the subtransient state, and the saliency of the rotor will be included in the model. In the case of one damper winding in each axis, the fast effects of the saturation are described. If damper windings in the d- and q-axis are included in the model, will look like Equations (5.44)-(5.47). There is no additional damping torque, because damper windings are included in the model.

5.5.1 Instantaneous value model

Implementing this in the machine equations for the Type 3A model, the instantaneous value Type 2A model, will look like Equations (5.44)-(5.47). There is no additional damping torque, because damper windings are included in the model.

\[
\begin{align*}
\psi_d &= \begin{bmatrix} -x_d & 0 & 0 & 1 & 1 & 0 \\ 0 & -x_q & 0 & 0 & 0 & 1 \\ 0 & 0 & -x_0 & 0 & 0 & 0 \\ x_d & 0 & 0 & 1 & \frac{x_{ad}}{x_f} & 0 \\ x_f & 0 & 0 & 1 & \frac{x_{ad}}{x_f} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_0 \\ i_d \\ i_f \\ i_{id} \end{bmatrix} \\
\psi_{id} &= \begin{bmatrix} -x_d & 0 & 0 & 1 & 1 & 0 \\ 0 & -x_q & 0 & 0 & 0 & 1 \\ 0 & 0 & -x_0 & 0 & 0 & 0 \\ x_d & 0 & 0 & 1 & \frac{x_{ad}}{x_{id}} & 1 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_0 \\ i_d \\ i_{id} \end{bmatrix} \\
\psi_{iq} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_0 \\ i_d \\ i_{id} \end{bmatrix} \\
\psi_{id} &= \begin{bmatrix} -x_d & 0 & 0 & 1 & 1 & 0 \\ 0 & -x_q & 0 & 0 & 0 & 1 \\ 0 & 0 & -x_0 & 0 & 0 & 0 \\ x_d & 0 & 0 & 1 & \frac{x_{ad}}{x_{id}} & 1 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_0 \\ i_d \\ i_{id} \end{bmatrix} \\
\psi_{iq} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_0 \\ i_d \\ i_{id} \end{bmatrix} \\
\end{align*}
\]

(5.44)

\[
\begin{align*}
\frac{du_d}{dt} &= -r_d u_d + \frac{1}{\alpha_n} \psi_d + \frac{1}{\alpha_n} \psi_0 + \frac{1}{\tau_f} \frac{d\psi_d}{dt} \\
\frac{du_q}{dt} &= -r_d u_q + \frac{1}{\alpha_n} \psi_q + \frac{1}{\tau_f} \frac{d\psi_q}{dt} \\
\frac{du_0}{dt} &= -r_d u_0 + \frac{1}{\alpha_n} \psi_0 + \frac{1}{\tau_f} \frac{d\psi_0}{dt} \\
\end{align*}
\]

(5.45)

\[
2H \frac{d\Delta \omega}{dt} = m_m - m_e
\]

(5.46)

\[
\frac{d}{dt} \delta = \Delta \omega
\]

(5.47)

There are two more state variables for this model than for the Type 3A model, \( \psi_{id} \) and \( \psi_{iq} \), i.e. the Type 2A instantaneous value model is an 8th order model.

5.5.1.1 Equivalent circuit

To be able to draw an equivalent circuit, the flux linkage and voltage equations are also stated in SI-units, see Equations (5.48 and 5.49).
Detailed Description of Synchronous Machine Models Used in Simpow

\[
\begin{bmatrix}
\Psi_d \\
\Psi_q \\
\Psi_0 \\
\Psi_f \\
\Psi_{id} \\
\Psi_{iq}
\end{bmatrix} = 
\begin{bmatrix}
-L_d & 0 & 0 & L_{ad} & L_{ad} & 0 \\
0 & -L_q & 0 & 0 & 0 & L_{aq} \\
0 & 0 & -L_0 & 0 & 0 & 0 \\
-L_{ad} & 0 & 0 & L_f & L_{ad} & 0 \\
-L_{ad} & 0 & 0 & L_{ad} & L_{id} & 0 \\
0 & -L_{aq} & 0 & 0 & 0 & L_{iq}
\end{bmatrix}
\begin{bmatrix}
I_d \\
I_q \\
I_0 \\
I_f \\
I_{id} \\
I_{iq}
\end{bmatrix}
\]

(5.48)

\[
\begin{bmatrix}
U_d \\
U_q \\
U_0 \\
U_f \\
U_{id} \\
U_{iq}
\end{bmatrix} = 
\begin{bmatrix}
-R_a & 0 & 0 & 0 & 0 & 0 \\
0 & -R_a & 0 & 0 & 0 & 0 \\
0 & 0 & -R_f & 0 & 0 & 0 \\
0 & 0 & 0 & R_{id} & 0 & 0 \\
0 & 0 & 0 & 0 & R_{iq} & 0 \\
0 & 0 & 0 & 0 & 0 & R_{iq}
\end{bmatrix}
\begin{bmatrix}
I_d \\
I_q \\
I_0 \\
I_f \\
I_{id} \\
I_{iq}
\end{bmatrix}
+ \frac{d}{dt}
\begin{bmatrix}
\Psi_d \\
\Psi_q \\
\Psi_0 \\
\Psi_f \\
\Psi_{id} \\
\Psi_{iq}
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\omega
\]

(5.49)

Where the upper-case letters are used when relating to quantities in SI units.

The equivalent \(d\)-and \(q\)-circuits for the instantaneous value model is viewed, using the leakage inductances, in Figure 5.9.

Figure 5.9 Equivalent \(dq\)-circuits for Type2A, leakage inductance instantaneous value model in SI units

The equivalent \(\theta\)-circuit for Type 2A is viewed in Figure 5.3.

### 5.5.2 Fundamental frequency model

From the set of equations for the instantaneous value model, see Equations (5.40–5.43), and implementing the changes stated in chapter 5.1, the fundamental frequency model is derived, see Equations (5.50–5.54).
5.6 Type 1A

In this chapter, the synchronous machine model Type 1A is described. This model is a simplification of Type 1, where the saturation effect is included. In chapter 5.7, the effects of the saturation are described.

If one extra damper winding is added in the \(q\)-axis, the accuracy in modelling solid iron rotor generators will be increased. Solid iron rotor is often used in large steam turbine generators, and provides multiple paths for circulating eddy currents. [9], [1] and [12]. This model is called Type1 in Simpow.

Additional damper windings may be included both in the \(d\)- and \(q\)-axis, see [4] and [5], this further increases the accuracy in the models.

5.6.1 Instantaneous value model

The extra \(q\)-damper winding changes the equations from the instantaneous value model Type 2 to:

\[
\begin{align*}
\frac{d}{dt} [x_d, x_q, \omega] &= [0, 0, 0] \\
\frac{d\delta}{dt} &= \Delta \omega
\end{align*}
\]
### Detailed Description of Synchronous Machine Models Used in Simpow

#### 5.6.1.1 Equivalent circuit

As for the other models, the equivalent circuit is made from the SI-units equations,

\[
\begin{bmatrix}
\psi_d \\
\psi_q \\
\psi_o \\
\psi_f \\
\psi_{ld} \\
\psi_{lq}
\end{bmatrix} = \begin{bmatrix}
-x_d & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & -x_q & 0 & 0 & 0 & 1 & 1 \\
-x_{ad} & 0 & 0 & 1 & \frac{x_{ad}}{x_f} & 0 & 0 \\
x_f & x_{ad} & 0 & 0 & 1 & 0 & 0 \\
x_{id} & 0 & 0 & 0 & 0 & 1 & 0 \\
x_{qd} & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix} \begin{bmatrix}
i_d \\
i_q \\
i_o \\
i_f \\
i_{ld} \\
i_{lq}
\end{bmatrix}
\]

(5.55)

\[
\left[ \begin{array}{c}
u_d \\
u_q \\
u_o \\
u_f \\
u_{ld} \\
u_{lq}
\end{array} \right] = \left[ \begin{array}{c}
r_a \\
r_a \\
r_a \\
r_a \\
r_a \\
r_a
\end{array} \right] + \left[ \begin{array}{c}
i_d \\
i_q \\
i_o \\
i_f \\
i_{ld} \\
i_{lq}
\end{array} \right]
\]

\[
\left[ \begin{array}{c}
\psi_d \\
\psi_q \\
\psi_o \\
\psi_f \\
\psi_{ld} \\
\psi_{lq}
\end{array} \right] = \left[ \begin{array}{c}
L_d \\
L_q \\
L_o \\
L_f \\
L_{ld} \\
L_{lq}
\end{array} \right] \left[ \begin{array}{c}
I_d \\
I_q \\
I_o \\
I_f \\
I_{ld} \\
I_{lq}
\end{array} \right] + \left[ \begin{array}{c}
i_d \\
i_q \\
i_o \\
i_f \\
i_{ld} \\
i_{lq}
\end{array} \right]
\]

\[
\left[ \begin{array}{c}
\psi_d \\
\psi_q \\
\psi_o \\
\psi_f \\
\psi_{ld} \\
\psi_{lq}
\end{array} \right] = \left[ \begin{array}{c}
L_d \\
L_q \\
L_o \\
L_f \\
L_{ld} \\
L_{lq}
\end{array} \right] \left[ \begin{array}{c}
I_d \\
I_q \\
I_o \\
I_f \\
I_{ld} \\
I_{lq}
\end{array} \right] + \left[ \begin{array}{c}
i_d \\
i_q \\
i_o \\
i_f \\
i_{ld} \\
i_{lq}
\end{array} \right]
\]

(5.59)

\[
\begin{align*}
2H \frac{d\Delta \omega}{dt} &= m_m - m_e \\
\frac{d}{dt} \delta &= \Delta \omega
\end{align*}
\]

(5.57)

(5.58)

There are one more state variable for this model than for the Type 2A model, \(\psi_{2q}\), i.e. the Type 1A instantaneous value model is a 9th order model.

#### 5.6.1.1 Equivalent circuit

As for the other models, the equivalent circuit is made from the SI-units equations, see Equations (5.55) and (5.56).

\[
\begin{bmatrix}
\psi_d \\
\psi_q \\
\psi_o \\
\psi_f \\
\psi_{ld} \\
\psi_{lq}
\end{bmatrix} = \begin{bmatrix}
-L_d & 0 & 0 & L_{ad} & L_{ad} & 0 & 0 \\
0 & -L_q & 0 & 0 & 0 & L_{aq} & L_{aq} \\
0 & 0 & -L_o & 0 & 0 & 0 & 0 \\
-L_{ad} & 0 & 0 & -L_f & 0 & 0 & 0 \\
0 & -L_{aq} & 0 & 0 & 0 & L_{aq} & L_{aq} \\
0 & -L_{aq} & 0 & 0 & 0 & 0 & L_{2q}
\end{bmatrix} \begin{bmatrix}
I_d \\
I_q \\
I_o \\
I_f \\
I_{ld} \\
I_{lq}
\end{bmatrix}
\]

(5.60)

Where the upper-case letters are used when relating to quantities in SI units.
The equivalent circuits for the instantaneous value model can be viewed using the leakage inductances, see Figure 5.10. This model is in Simpow referred to as synchronous machine model Type 1A.

\[
\begin{align*}
\dot{\psi}_d &= -x_d \psi_d + \frac{\omega - \delta}{\tau} \psi_{1d} + 0 \psi_{2d} \\
\dot{\psi}_q &= -x_q \psi_q + \frac{\omega - \delta}{\tau} \psi_{1q} + 0 \psi_{2q} \\
\dot{\psi}_f &= -x_{ad} \psi_f + x_{ad} \psi_q \\
\dot{\psi}_l &= -x_{ld} \psi_l + x_{ld} \psi_d \\
\dot{\psi}_q &= -x_{aq} \psi_q + x_{aq} \psi_d \\
\dot{\psi}_2 &= -x_{2q} \psi_2 + x_{2q} \psi_d \\
\end{align*}
\]

\[
\begin{bmatrix}
\dot{u}_d \\
\dot{u}_q \\
\dot{u}_f \\
\end{bmatrix}
= 
\begin{bmatrix}
-x_d & 0 & 1 & 1 & 0 & 0 \\
0 & -x_q & 0 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\dot{i}_d \\
\dot{i}_q \\
\dot{i}_f \\
\dot{i}_{1d} \\
\dot{i}_{1q} \\
\end{bmatrix}
+ 
\begin{bmatrix}
\tau_f \\
\tau_{1d} \\
\tau_{1q} \\
\tau_{2q} \\
\end{bmatrix}
\begin{bmatrix}
\psi_f \\
\psi_{1d} \\
\psi_{1q} \\
\psi_{2q} \\
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
+ 
\begin{bmatrix}
\omega \\
0 \\
0 \\
0 \\
\end{bmatrix}
\begin{bmatrix}
\psi_d \\
\psi_q \\
\psi_f \\
\psi_l \\
\psi_2 \\
\end{bmatrix}
\]

\[
2H \frac{d\Delta \omega}{dt} = m_m - m_e 
\]  

\[
\frac{d\delta}{dt} = \Delta \omega 
\]

with
In the same manner as for the instantaneous value model, it is verified that this model has one more state variable than the Type 2A model. Thus, the Type 1A fundamental frequency model is of order 6.

5.7 Saturation

Models of Type 1-3 are modelled both with and without consideration of the saturation in the d- and q-axis. In the previous chapters, models of Type1A-3A are described, i.e. models were the saturation effect is excluded.

As the field current exceeds a certain level, a non-linear relationship occurs between the flux linkage and the current [9], i.e. between the flux linkage and the magneto motive force. An example of this effect can be seen in Figure 5.11. When this level is exceeded, the flux linkages will be smaller for a model where saturation is included than for a model where saturation is not taken under consideration.

Figure 5.11 Flux linkage - mmf diagram, showing effects of saturation

There are several different ways to model this saturation effect. In Simpow, the saturation function can be modelled as a linear relation between the saturation factor and the flux linkage, \( S(\psi) \), or by using a self-designed saturation table with corresponding values of the magneto motive force, \( W \), and the saturation function, \( f(W) \), as plotted in the figure above. While in e.g. the power system simulation software, PSS/E, either quadratic or exponential saturation can be modelled, \( S(\psi) \).

There are different opinions which of the models that is the most realistic, however, the differences in the outcome are relatively small [18].

Due to the non-linearity, the flux linkage equations will include the saturation function, as described in Equation (5.66).
Where

\[ W_d = i_f + i_{id} - i_d x_{ad} \]
\[ W_q = i_{iq} + i_{2q} - i_q x_{aq} \]  \hspace{1cm} (5.67)

\( W_d, W_q \) are the magneto motive forces in the \( d \)-axis and \( q \)-axis.

\( f_d(W_d), f_q(W_q) \) are the saturation functions for the \( d \)-axis and \( q \)-axis.

If \( f_d(W_d) = W_d, f_q(W_q) = W_q \), then no saturation is included, and the Equation (5.66) will be the same as the flux linkage equations described in sections 5.4-5.6.

For the models with fewer windings, the rows and columns of the matrix, corresponding to the windings that are not included, are to be neglected.

In the manner that Equation (5.66) is written, it is easy to realise that the difference between the flux linkage and the mmf, is a leakage flux depending on the leakage reactance and the current. It can also be seen that there are no forces acting in the \( \theta \)-axis direction.
Programming

Dr. Kjell Anerud, at ABB Utilities department Power Systems Analysis, has developed the Dynamic Simulation Language used in Simpow. In this chapter, a short description of the DSL-programming is made, since it is a far too complicated matter to be detailed described in this report.

When a power system is simulated in Simpow, a DSL-file can be used when modelling different components of the system, an example of such a component is a regulator. When calling the DSL-file, model specific data of the component has to be given by the user, e.g. rated power, rated voltage, etc.

When a DSL-file is used when modelling a synchronous machine, system specific data is sent to the DSL-file, which in return gives the machine response. Inside the DSL-file, the machine equations derive the characteristics of the machine. An example of how these equations are defined in the DSL-code, is given below.

\[ (TMX+TE)=2.*H*.D/DT.WR \]

DSL-code: Expression from the Type 3, synchronous machine model.

The first part of this expression, defines \( WR \) as the variable to be derived from the torque equation:

\[ (TMX+TE)=2.*H*.D/DT.WR \]

DSL-code: Torque equation from the Type 3, synchronous machine model.

Here, the following definitions has been made:

- \( TMX \) is the per unit value of the mechanical torque applied on the machine axis, which in this report has been called \( m_m \).
- \( TE \) is the per unit value of the electrical torque, which in this report has been called \( m_e \) and has the opposite sign.
- \( H \) is the per unit inertia constant, and is one of the model specific data of the synchronous machine, that has to be given by the user.
- \( .D/DT. \) is the Laplace-operator, i.e. \( d/dt \) in this report.
- \( WR \) is the per unit value of the machine rotor speed, which in this report has been called \( \omega/\omega_n \).

In Simpow, it is possible to choose if the output, \( TM \), from the turbine is mechanical power or mechanical torque, why the mechanical torque, \( TMX \), has to be calculated as:

\[
\begin{align*}
\text{IF}(\text{ISP.EQ.1.AND.WR.NE.0.}) & \text{THEN} \\
TMX &= TM/WR \\
\text{ELSE} & \\
TMX &= TM \\
\text{ENDIF}
\end{align*}
\]

DSL-code: If statement from the Type 3, synchronous machine model.

Here the if-statement, \((\text{ISP.EQ.1.AND.WR.NE.0.})\), has the same meaning as \((\text{ISP}=1 \& \text{WR} \neq 0)\). Where \( ISP \) has the value 0, if the output from the turbine is mechanical torque, and the value 1, if the output from the turbine is mechanical power. I.e. the mechanical torque of the synchronous machine, \( TMX \), is equal to
If the output from the turbine is mechanical torque, or if the machine is at standstill. Otherwise, \( \text{TMX} \) is equal to the fraction \( \text{TM} / \text{WR} \).

The mechanical torque of the synchronous machine is also an output variable from the machine, which can be plotted by the user. In this case it is called \( \text{MT} \) and is on the per unit base \( S_n/\omega_n \), i.e. to get the real per unit value of the machine in question, \( \text{MT} \) must be divided by the pole pair number of the machine.

Four different DSL-files have been created, modelling the different synchronous machines described in the previous chapters. These are named: Type1A.dsl, Type2A.dsl, Type3A.dsl and Type4.dsl, and are containing the fundamental frequency models of the same Type as the name of the files, where the ‘A’ stands for that no saturation is included in these models.

This was a short description of the structure of DSL, and of the content of the DSL-files developed during this thesis. Further comments can be seen in the code in the different programmes.

For a more detailed description of the Dynamic Simulation Language used in Simpow, see the Simpow manual, [3].
7 Validation

When trying to validate the DSL coded machine models, the old FORTRAN models have stood as the reference. A small power system has been used for this purpose, consisting of: a synchronous machine, a line and a swing-bus, see Figure 7.1.

In Simpow it is possible to include numerous independent power systems in the same run, therefore two mirrored power systems have been simulated. All characteristics of the systems are the same; the only difference is the machine model. In one of the systems, a DSL-coded machine model is used, and in the other, a FORTRAN-coded model is used. During simulations, two different disturbances has been added to the system, a three-phase to ground fault and a one-phase to ground fault. In this way, both symmetrical and unsymmetrical conditions of the system are monitored. Various outputs can be viewed from the models, a list of variables that can be plotted for the existing synchronous machine models is attached in and in Appendix A.

During the simulations, specific data for the system and for the models has been used; these in data files can be viewed in Appendix B.

To view the differences between the DSL- and FORTRAN-coded models, various outputs have been compared. In Figure 7.2, the direct axis voltage is plotted for both the DSL- and the FORTRAN-coded Type 1A model.
Figure 7.2 Ud for both the DSL- and FORTRAN-coded Type 1A during and after a three-phase fault at the generator node

Obviously, no apparent difference can be seen; therefore, the differences between the voltages are plotted in Figure 7.3.

Figure 7.3 Difference in Ud between DSL and FORTRAN-models during and after a three-phase fault at the generator node

The maximum difference between the models are about 0.00045 p.u., this creates a maximum deviation of about 0.065 %, hence, a rather small difference. In Table 7.1, the differences of the currents, voltages and speed are assembled.
Table 7.1 Assembly of differences between the DSL- and for the FORTRAN-coded Type 1A
for a three-phase fault at the generator node

<table>
<thead>
<tr>
<th></th>
<th>A: maximum difference</th>
<th>B: minimum value while maximum difference</th>
<th>C=A/B: maximum deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_d$</td>
<td>0.010293</td>
<td>0.777</td>
<td>1.32%</td>
</tr>
<tr>
<td>$i_q$</td>
<td>0.00085</td>
<td>0.4123</td>
<td>0.21%</td>
</tr>
<tr>
<td>$u_d$</td>
<td>0.00044</td>
<td>0.718</td>
<td>0.06%</td>
</tr>
<tr>
<td>$u_q$</td>
<td>0.0032</td>
<td>0.607</td>
<td>0.53%</td>
</tr>
<tr>
<td>$\omega/\omega_n$</td>
<td>0.000162</td>
<td>0.998</td>
<td>0.02%</td>
</tr>
</tbody>
</table>

Table 7.1 Assembly of differences between the DSL- and FORTRAN-coded Type 1A for a three-phase fault at the generator node

In Table 7.2, the eigenvalues are assembled for the different machine models. The indices A and B refer to the corresponding eigenvalues of the DSL and FORTRAN models, and obviously, they do not have any significant difference.

<table>
<thead>
<tr>
<th></th>
<th>Type1A</th>
<th>Type2A</th>
<th>Type3A</th>
<th>Type4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$-A</td>
<td>(-0.189615, 1.021520)</td>
<td>(-0.186307, 1.016390)</td>
<td>(-0.120430, 1.010864)</td>
<td>(0, 1.011586)</td>
</tr>
<tr>
<td>$\lambda_1$-B</td>
<td>(-0.189615, 1.021520)</td>
<td>(-0.186307, 1.016390)</td>
<td>(-0.120430, 1.010864)</td>
<td>(0, 1.011586)</td>
</tr>
<tr>
<td>$\lambda_2$-A</td>
<td>(-0.189616, -1.021520)</td>
<td>(-0.186307, -1.016390)</td>
<td>(-0.120430, -1.010864)</td>
<td>(0, -1.011586)</td>
</tr>
<tr>
<td>$\lambda_2$-B</td>
<td>(-0.189616, -1.021520)</td>
<td>(-0.186307, -1.016390)</td>
<td>(-0.120430, -1.010864)</td>
<td>(0, -1.011586)</td>
</tr>
<tr>
<td>$\lambda_3$-A</td>
<td>(-0.051660, 0.000000)</td>
<td>(-36.428078, 0.000000)</td>
<td>(-0.051662, 0.000000)</td>
<td>(-0.051662, 0.000000)</td>
</tr>
<tr>
<td>$\lambda_3$-B</td>
<td>(-0.051660, 0.000000)</td>
<td>(-36.428085, 0.000000)</td>
<td>(-0.051662, 0.000000)</td>
<td>(-0.051662, 0.000000)</td>
</tr>
<tr>
<td>$\lambda_4$-A</td>
<td>(-2.145103, 0.000000)</td>
<td>(-36.428083, 0.000000)</td>
<td>(-36.428097, 0.000000)</td>
<td>(-36.428097, 0.000000)</td>
</tr>
<tr>
<td>$\lambda_4$-B</td>
<td>(-2.145103, 0.000000)</td>
<td>(-36.428097, 0.000000)</td>
<td>(-36.428097, 0.000000)</td>
<td>(-36.428097, 0.000000)</td>
</tr>
<tr>
<td>$\lambda_5$-A</td>
<td>(-22.065731, 0.000000)</td>
<td>(-38.227016, 0.000000)</td>
<td>(-38.227016, 0.000000)</td>
<td>(-38.227016, 0.000000)</td>
</tr>
<tr>
<td>$\lambda_5$-B</td>
<td>(-22.065739, 0.000000)</td>
<td>(-38.227020, 0.000000)</td>
<td>(-38.227020, 0.000000)</td>
<td>(-38.227020, 0.000000)</td>
</tr>
<tr>
<td>$\lambda_6$-A</td>
<td>(-22.065731, 0.000000)</td>
<td>(-38.227016, 0.000000)</td>
<td>(-38.227016, 0.000000)</td>
<td>(-38.227016, 0.000000)</td>
</tr>
<tr>
<td>$\lambda_6$-B</td>
<td>(-22.065739, 0.000000)</td>
<td>(-38.227020, 0.000000)</td>
<td>(-38.227020, 0.000000)</td>
<td>(-38.227020, 0.000000)</td>
</tr>
</tbody>
</table>

Table 7.2 Eigenvalues of the power system, for the different machine models

From the tables and the plots, it is obvious that the differences between the DSL-written models and the FORTRAN models are small. Therefore, the conclusion drawn is that there is a strong resemblance between the models, although further simulations should be made to fully state the validity of the DSL-models. In Appendix C the DSL-file for the machine model Type 1A is attached, and in Appendix D a block diagram of the model is viewed.
8 Conclusions and future work

8.1 Conclusions

The following conclusions are made:

- It has been found that the equations summarised in the technical reports [1] and [2], describing the underlying equations of the synchronous machine models in Simpow, have theoretical foundations. References to [1] and [2], have been found in Bühler, [10], and Laible, [13], after a vast literature research. Apart from this, a recommended source of information is Kundur [9], although he has a different reference system.

- The synchronous machine models have been documented and explained in detail in this technical report, including thoroughly derivation of the equations with references to different sources in the literature.

- The four synchronous machine fundamental frequency models from Simpow have successfully been realised with DSL. Although further validation is needed, the simulations made show a strong resemblance with the already existing FORTRAN models.

It is unnecessary to use the simple and less correct models, Type 3 and 4, in a simulation, just to save computer time. With today’s computers, the more accurate models, Type 1 and 2, could be used with extended reliability, and without noticeable extension of the simulation time.

8.2 Future work

In future work, the following paragraphs could be considered:

- Including saturation in the DSL-coded models, regarding the parameters: DTAB, QTAB, V1D, V2D, SE1D, SE2D, V2Q, SE1Q, SE2Q, TAB, V1, V2, SE1 and SE2, see the Simpow users manual [3].

- Including the following parameters, implemented in the already existing models, see also the Simpow users manual [3].

  CNODE - for control of the machine from another node.
  PF, QF - for generation/consumption of only a part of the power at the node.
  NCON, TET0, F0, U0 - for modelling of a machine not connected to the system.
  VREG, TURB, XTURB, LOAD, 1BREAKER - identification numbers for the regulator, turbines, load and breaker associated with the machine.
  FREEZE - for freezing the state of the machine when it is disconnected.
  REXP - for frequency dependent resistance.

- Further validation of the DSL models.
- Possibly, validation of the FORTRAN models.
- Realisation of the instantaneous value models in DSL.
- Updating the Simpow user manual.
Appendix A

List of variables that can be plotted for the existing synchronous machine models, by Tore Petersson

September 25, 2001/ T.P.

Dynpost: Synchronous machines

Variables which can be plotted for the synchronous machine models of type 1, 1A, 2, 2A, 3, 3A and 4

General information

For synchronous machine models of type 1 and 1A all the listed variables can be plotted. For the other models listed above, the damping-circuit variables available are dependent on the type; reference is made to the type descriptions in the Dynpow part of the SIMPOW manual.

Furthermore, for type 4 no field-circuit variable can be plotted.

Per unit base values

In defining the per unit (p.u.) base values in the succeeding subsections, the following parameters are used:

\[ S_{\text{netw}} = \text{The network three-phase base power in MVA, i.e. the specified SN value in Optpow/General} \]

\[ U_{\text{netw}} = \text{The machine node phase-to-phase base voltage, in kV RMS, i.e. the specified UB value in Optpow/Nodes. In usual cases where the machine is connected to a step-up transformer and with respect to facilitate interpreting simulation results of some variables it is recommended to choose UB equal to the kV rating UN of the machine.} \]

\[ I_{\text{netw}} = \frac{S_{\text{netw}}}{\sqrt{3} U_{\text{netw}}}, \text{in kA RMS} \]

\[ S_n = \text{The machine MVA rating, i.e. the specified SN value in Dynpow/Synchronous machines} \]

\[ U_n = \text{The machine kV RMS rating, i.e. the specified UN value in Dynpow/Synchronous machines} \]

\[ I_n = \frac{S_n}{\sqrt{3} U_n}, \text{the machine kA RMS rating, i.e.} \]
\( \omega_n = \) The nominal angular frequency of the machine and the power system, in el. rad/s, i.e. usually \( 2\pi 50 \) or \( 2\pi 60 \) el. rad/s

\( \omega_{mech\ n} = \) The nominal angular rotor speed of the machine considered, in mech. rad/s

The above listed synchronous machine models use a so-called non-reciprocal rotor p.u. system.

The p.u. base values of the field-circuit plot variables \( IF \) and \( UF \) are defined as follows:

- One p.u. field current is the field current which would theoretically be required to produce one p.u. stator voltage, i.e. rated voltage, on the air-gap line at open-circuit rated speed steady-state conditions.
- One p.u. field voltage is the corresponding field voltage at the field winding temperature to consider (usually 75 or 100 degrees centigrade).

The p.u. base values of the damper circuit currents \( IDD \), \( IQ1 \) and \( IQ2 \) are defined in the same way as for the field current \( IF \).

The p.u. base values of the “m.m.f.” plot variables \( WD \) and \( WQ \) and the magnetic flux linkages \( FID, FIQ, FIF, FIDD, FIQ1, FIQ2 \) are determined by the p.u. base values of the rotor currents and of the p.u. base values of the inductances.

**Sign conventions**

Generator or source sign convention is used, meaning that stator currents and powers are positive out of the machine. Positive torque and speed directions are that of normal rotor rotation.

**The torque variables MT, ME, MEP, MEN and the speed variable SPEED**

The p.u. base value \( S_n / \omega_n MNm \) is referred to the equivalent two-pole machine model used. The physical torque values in p.u. are the same but then on base \( S_n / \omega_{mech\ n} MNm \). Similarly SPEED is expressed in p.u. on \( \omega_n \) as referred to the equivalent two-pole machine model used. The physical rotor speed in p.u. is the same but then on base \( \omega_{mech\ n} \).

The plot variables are divided into three groups: Transta only, Masta only, Transta and Masta.

**Transta**

*Magnitude of stator voltages, in p.u. on base \( U_{new} / \sqrt{3} \text{ kV RMS (or in kV RMS)} \):

\[
\begin{align*}
U &= \text{Positive-sequence voltage} \\
UN &= \text{Negative-sequence voltage} \\
U0 &= \text{Zero-sequence voltage} \\
UA,UB,UC &= \text{Phase voltages}
\end{align*}
\]
**Magnitude of stator currents, in p.u. on base \( I_n \) kA RMS (or in kA RMS):**

- \( I \) = Positive-sequence current
- \( I_N \) = Negative-sequence current
- \( I_0 \) = Zero-sequence current
- \( I_A, I_B, I_C \) = Phase currents

- \( P \) = Positive-sequence active power output, in p.u. on base \( S_n \) MVA (or in MW)
- \( Q \) = Positive-sequence reactive power output, in p.u. on base \( S_n \) MVA (or in Mvar)

- \( MEP \) = Electromagnetic torque produced by positive-sequence stator currents, in p.u. on base \( S_n/\omega_n \) MNm
- \( MEN \) = Electromagnetic torque produced by negative-sequence stator currents, in p.u. on base \( S_n/\omega_n \) MNm

**Additional variables available for special purposes, for instance for analysis of relay protection matters:**

- Stator voltage components, in p.u. on base \( U_{netw}/\sqrt{3} \) kV RMS (or kV RMS) and referred to the common (global) real-imaginary reference frame:

  - \( UR, UI \) = Real and imaginary components of positive-sequence stator voltage
  - \( UNR, UNI \) = Real and imaginary components of negative-sequence stator voltage
  - \( U0R, U0I \) = Real and imaginary components of zero-sequence stator voltage

- Stator current components, in p.u. on base \( I_n \) kA RMS (or kA RMS) and referred to the common (global) real-imaginary reference frame:

  - \( IR, II \) = Real and imaginary components of positive-sequence stator current
  - \( INR, INI \) = Real and imaginary components of negative-sequence stator current
  - \( I0R, I0I \) = Real and imaginary components of zero-sequence stator current

**Equivalent impedance values as determined by the ratio of voltage phasor to current phasor \( (U/I) \) in p.u. on \( U_{netw}/(\sqrt{3} I_n) \) Ohm (or in Ohm):**

- \( ZA, ZB, ZC \) = Impedance (magnitude)
- \( RA, RB, RC \) = Resistance
- \( XA, XB, XC \) = Reactance
Detailed Description of Synchronous Machine Models Used in Simpow

**Sequence impedances**

- **Z, ZN, Z0** = Positive, negative and zero-sequence impedance (magnitude)
- **R, RN, R0** = Positive, negative and zero-sequence resistance
- **X, XN, X0** = Positive, negative and zero-sequence reactance

**Masta**

*Stator voltages, in p.u. on base* $\sqrt{2/3}$ $U_{\text{netw}}$ kV (or in kV):

- **U** = Voltage, calculated as $\sqrt{u_d^2 + u_q^2 + u_0^2}$
- **UZ** = “Zero-sequence” voltage ($u_0$)
- **UA, UB, UC** = Phase voltages

*Stator Currents, in p.u. on base* $\sqrt{2}$ $I_n$ kA (or in kA):

- **I** = Current, calculated as $\sqrt{i_d^2 + i_q^2 + i_0^2}$
- **IZ** = “Zero-sequence” current ($i_0$)
- **IA, IB, IC** = Phase currents

- **P** = Instantaneous active power output, calculated as $u_d i_d + u_q i_q + 2u_0 i_0$, in p.u. on base $S_n$, MVA (or in MW)
- **Q** = Instantaneous “Reactive power” output, calculated as $u_q i_d - u_d i_q$, in p.u. on base $S_n$, MVA (or in Mvar). It is a meaningful quantity only for steady-state balanced conditions.

**Transta and Masta**

- **IF** = Field current, in p.u (For p.u. base value, see General information above.)
- **UF** = Field voltage, in p.u. (For p.u. base value, see General information above.)

- **MT** = Mechanical torque, in p.u. on base $S_n / \omega_n$ MNm
- **ME** = Total electromagnetic torque, in p.u. on base $S_n / \omega_n$ MNm

- **SPEED** = Rotor speed, in p.u. on base $\omega_n$
- **TETA** = Rotor angle, in el. degrees (KVAR) or el. radians (VAR)

TETA is the difference angle between the q-axis of the machine considered and the q-axis of the reference machine as specified in Dynpow/General.
Additional variables available for detailed machine analysis:

**Stator variables** in p.u. on machine base values and referred to the local reference frame, i.e. the machine d-q reference frame:

*Stator voltages in p.u. on base $U_n/\sqrt{3}$ kV RMS in Transta and $\sqrt{2/3} U_n$ kV in Masta*

- **UDL** = Direct-axis stator voltage
- **UQL** = Quadrature-axis stator voltage

*Stator currents in p.u. on base $I_n$, kA RMS in Transta and $\sqrt{2} I_n$, kA in Masta*

- **IDL** = Direct-axis stator current
- **IQL** = Quadrature-axis stator current

**Other variables** in p.u. (For p.u. base values, see General information above.)

- **IDD** = Direct–axis damper circuit current of machines of type 1, 1A, 2 and 2A
- **IQ1** = Quadrature-axis damper circuit 1 current of machines of type 1, 1A, 2 and 2A
- **IQ2** = Quadrature-axis damper circuit 2 current of machines of type 1 and 1A
- **WD** = Variable to which the m.m.f. related to the direct-axis stator to rotor mutual inductance is proportional ($WD = IF + IDD - IDL* L_{ad}$)
- **WQ** = Variable to which the m.m.f. related to the quadrature-axis stator to rotor mutual inductance is proportional ($WQ = IQ1 + IQ2 - IQL* L_{aq}$)
- **FID** = Direct-axis stator flux linkages
- **FIQ** = Quadrature-axis stator flux linkages
- **FIF** = Field flux linkages
- **FIDD** = Direct-axis damper circuit flux linkages
- **FIQ1** = Quadrature-axis damper circuit 1 flux linkages
- **FIQ2** = Quadrature-axis damper circuit 2 flux linkages
Detailed Description of Synchronous Machine Models Used in Simpow
Appendix B

Simpow test case files

Optpow (Load flow) file

** DSL-FORTRAN.OPTPOW **

GENERAL
SN=2220
END

NODES
BUS1 UB=24
BUS2 UB=24
END

LINES
BUS1 BUS2 TYPE=11 X=0.65
END

POWER CONTROL
BUS1 TYPE=NODE RTYP=PQ P=1998 Q=666
BUS2 TYPE=NODE RTYP=SW U=23.88 FI=0
END

NODES
BUS11 UB=24
BUS12 UB=24
END

LINES
BUS11 BUS12 TYPE=11 X=0.65
END

POWER
BUS11 TYPE=NODE RTYP=PQ P=1998 Q=666
BUS12 TYPE=NODE RTYP=SW U=23.88 FI=0
END

END
Detailed Description of Synchronous Machine Models Used in Simpow

Dynpow (Dynamic simulation) file

evaluation of dsl models compared to fortran models
!two separate systems, each containing:
!one synchronous machine, one line and one swingbus
**
CONTROL DATA
TETL=1000
TEND 90 xtrace 1
DDSL=3
!!!! HMIN 0.00005
!!!! HMAX 0.0005
END
GENERAL
NERF 2
FN 60 60
REF BUS2 BUS12
END

NODES
BUS2 TYPE 1
BUS12 TYPE 1
END

SYNCHRONOUS MACHINE
dslgen BUS1 TYPE=DSL/TYPESA/ SN 2220 UN 24 H 3.5 RA 0.03
XD 1.81 XQ 1.76 XDP 0.3 XA 0.16 TD0P=0.03 TQ0B=0.07
R0=0.2 X0=0.8 R2=0.03 X2=0.2

fortran gen BUS11 TYPE=1A SN 2220 UN 24 H 3.5 RA 0.003
XD 1.81 XQ 1.76 XDP 0.3 XA 0.16 TD0P=0.03 TQ0B=0.07
R0=0.2 X0=0.8 R2=0.03 X2=0.2

END

DSL-TYPES
TYPE1A(BUS,SN,UN,H,D/0/,RT/0/,XT/0/,RA,XD,XQ,XDP,
XQ,B,XQ,B,TD0P,TQ0P,TDB,TDB,
U0,F0,P0,Q0,UF0,UF,VC,TM0,WM,TETR,ICON,
ISP/I:0/,SPE/I:1/, R0,X0,R2,X2)

TYPE2A(BUS,SN,UN,H,D/0/,RT/0/,XT/0/,RA,XD,XQ,XDP,
XQ,B,XQ,B,TD0P,TQ0B,TDB,TDB,
U0,F0,P0,Q0,UF0,UF,VC,TM0,WM,TETR,ICON,
ISP/I:0/,SPE/I:1/, R0,X0,R2,X2)

TYPE3A(BUS,SN,UN,H,D/0/,RT/0/,XT/0/,RA,XD,XQ,XDP,
RA,TD0P,
U0,F0,P0,Q0,UF0,UF,VC,TM0,WM,TETR,ICON,
ISP/I:0/,SPE/I:1/, R0,X0,R2,X2)

TYPE4(BUS,SN,UN,H,D/0/,RT/0/,XT/0/,RA,XD,XQ,XDP,
U0,F0,P0,Q0,UF0,UF,VC,TM0,WM,TETR,ICON,
ISP/I:0/, R0,X0,R2,X2)

_CONSTANT_FIELD(UF0,UF)
_CONSTANT_TORQUE(TM0,TM)
END

FAULT
F1 NODE=BUS1 TYPE=3PSG
F2 NODE=BUS1 TYPE=1PSG
F11 NODE=BUS11 TYPE=3PSG
F12 NODE=BUS11 TYPE=1PSG
END

RUN INSTRUCTION
IAT 0.1 INST CONNECT FAULT F1
IAT 0.1 INST CONNECT FAULT F11
IAT 0.15 INST DISCONNECT FAULT F1
IAT 0.15 INST DISCONNECT FAULT F11
AT 0.1 INST CONNECT FAULT F2
AT 0.1 INST CONNECT FAULT F12
AT 0.15 INST DISCONNECT FAULT F2
AT 0.15 INST DISCONNECT FAULT F12

END
Appendix C

DSL-file for synchronous machine model Type 1A.

MODEL with one field winding and one d- and two q-axis damperwindings

PROCESS TYPE1A(NODE1, SN, UN, H, D, RT, XT, RL, XD, XQ, XDP,
 & XQP, XDB, XQB, XL, TD0P, TD0B, TQ0P, TQ0B, TQ0B,
 & U0, FI0, P0, Q0, UF, UC, MEC0, MEC0, SPEED, TETR, ICON,
 & ISP, SPE, R0, X0, R2, X2)
EXTERNAL SN, UN, H, D, RT, XT, RL, XD, XQ, XDP, XQP
EXTERNAL XDP, XQP, XL, TD0P, TD0B, TQ0P, TQ0B, TQ0B
EXTERNAL U0, FI0, P0, Q0, UF
EXTERNAL MEC0, TETR, ISP, SPE
EXTERNAL R0, X0, R2, X2
INTEGER ICON, ISP, SPE
REAL VC, UF0/*/, MECH0/*/, TETR, XQP, XDP, TQ0B, TQ0P, TM
REAL U0, FI0, P0, Q0, TD0B, XDB
REAL U2, PX, QX, FAC1, FAC2
REAL SN, UN, H, D, RT, XT, RL, XD, XQ, XDP, TD0P, UF, MECH
REAL R0, X0, R2, X2
AC NODE1
AC_CURRENT I/NODE1/
REAL RA/*/, XA/*/, XAD/*/, XAQ/*/, XRF/*/, TF/*/, UB/*/, W0/*/
REAL SIGMAFD/*/, TD/*/, XDD/*/, RF/*/
REAL UBASE/*/, IBASE/*/, SIGMA1Q2/*/, XQ1/*/, XQ2/*/, TQ1/*/, TQ2/*/
REAL UPRE, UPIM, SINT, COST, PSID, PSIQ, IP, TE, PSI1D, PSI1Q, PSI2Q
REAL UD, UQ, WD, WQ, PWD, PQW, UDX/*/, UQX/*/
REAL G2/*/, B2/*/, B0/*/, R0/*/, X0/*/, B0/*/, BO0, Q00, IN2, TE2
REAL TETA, SPEED, PSI1P, IP, TD, IQ, IDD, I0, IQ0, IQ2, TETAI, PI
STATE TETA, SPEED, PSI1P, IP, TD, IQ, IDD, I0, IQ0, IQ2, TETAI, PI
STATE TETAI, PI, ICON/0/, WD, WQ
PLOT IDL/ID/, IQL/IQ/, SPEED, TE2, TD, UD, UQ, TETA
PLOT ID, IQ, I0, IQ2, IQ2, WD, WQ

IF (ISP.EQ.1) THEN !i.e. if mech=mechanical power (in DSL_TYPE: TM)
  TM=MECH/SPEED
ELSE !i.e. if mech=mechanical torque (in DSL_TYPE: TM)
  TM=MECH
ENDIF

IF (START) THEN
  !negative and zero sequences
  QQ=R2**2*R2+X2**2
  IF(QQ.EQ.0) THEN
    G2=0.
    B2=0.
  ELSE
    G2=R2/2/QQ
    B2=-X2/QQ
  ENDIF
  QQ=RO**2+RO**2
  IF(QQ.EQ.0) THEN
    G0=0.
    B0=0.
  ELSE
    G0=R0/2/QQ
    B0=-X0/2/QQ
  ENDIF

  !armature resistance and leakage reactance
  RA=RL+RT
  XA=XL+XT

  !reactance definitions
  XAD=XD-XA
  XAQ=XQ-XA
  XRF=XAD**2/(XD-XDP)
  UBASE=UBASE(NODE1) !machine node base voltage
  UBASE=UBASE/UN !global-local voltage transformation factor
  W0=-D/DT.F10(NODE1)
  IBASE=UBASE*SN/PBASE !global-local current transformation factor
  SPEED=1

  !definitions of the initial conditions
  FAC1=RA**2+XQ**2
  U2=U0**2
  PX=P0/U2
  QX=Q0/U2
  FAC2=1.-(FX**2+QX**2)*FAC1+2.*(PX**2+QX**2)
  FAC2=SQRT(FAC2)

  !first initial voltage and current
  IF(START00) THEN
UDX=U0*(PX*XQ-QX*RA)/FAC2
UQX=U0*(1.+PX*RA+QX*XQ)/FAC2
ID=PX*UDX+QX*UQX
IQ=PX*UQX-QX*UDX
ELSE
PX=ID*UDX+IQ*UQX
QX=ID*UQX-ID*UDX
ID:PX-P0=0
IQ:QX-Q0=0
UDX:SQRT(UDX**2+UQX**2)-U0=0
UQX:WQ+XAQ*IQ=0.
ENDIF

! dq-components of the flux linkage
IF (SPE.EQ.1) THEN
PSID=(UQX+RA*IQ)/SPEED
PSIQ=-(UDX+RA*ID)/SPEED
ELSE
PSID=(UQX+RA*IQ)
PSIQ=-(UDX+RA*ID)
ENDIF

! mmf in the d resp q axis
FWD=PSID+ID*XA
FWQ=PSIQ+IQ*XA
!no saturation
WD=FWD
WQ=FWQ

IF (DISC.AND.(UQX.NE.0.OR.UDX.NE.0)) THEN
TETAI=ATAN2(UDX,UQX)
ELSE
TETAI: UQX*SIN(TETAI)=UDX*COS(TETAI)
ENDIF

! external load angle
TETA=TETAI+FI0
SINT=SIN(TETA)
COST=COS(TETA)

! field current, resistance, voltage
IF=WD+ID*XAD
RP=(XAD*XRF)/TD0P/W0
UFO=IF
! field flux linkage
PSIF=IF*(1-XAD/XRF)*XAD/XRF*FWD

! mechanical power or torque (in DSL_TYPE: TM0)
! TR NKH p.12 or Kundur p.100
IF (ISP.EQ.1) THEN
MECH0=(PSID*IQ-PSIQ*ID)/SPEED
ELSE
MECH0=PSID*IQ-PSIQ*ID
ENDIF

! TYPE 1A-VARIABLES:
! reactance of winding in d-axis
XDD=XAD+(XDP-XA)*(XDB-XA)/(XDP-XDB)
! field winding decrement factor
SIGMAFD=1-XAD**2/XRF/XDD
! time constant of d-damper winding
TD=TD0P/SIGMAFD

! time constant of field winding
Tp=TD0P-TD
! reactance of 1:st q-damper winding
XQ1=XAQ**2/(XQ-XQP)
! reactance of 2:nd q-damper winding
XQ2=XAQ*(XQP-XA)/(XQP-XQB)
! q-damper windings decrement factor
SIGMAQ1Q2=1-XAQ**2/XQ1/XQ2
! time constant of 2:nd q-damper winding
TQ2=TQ0B/SIGMAQ1Q2
! time constant of 1:st q-damper winding
TQ1=TQ0P-TQ2

! flux linkages of the damper windings
PSIDD=XAD/XDD*FWD
PSIQ1=XAQ/XQ1*FWQ
PSIQ2=XAQ/XQ2*FWQ

! real-/imag. voltage in machine per unit
UPRE=UPRE(NODE1)*UBASE
UFIM=UFIM(NODE1)*UBASE
ELSE
! else if(start)
! real-/imag. voltage in machine per unit
UPRE=UPRE(NODE1)*UBASE
UFIM=UFIM(NODE1)*UBASE
! voltage angle

! field flux linkage
PSIF=IF*(1-XAD/XRF)*XAD/XRF*FWD
SINT=SIN(TETA)
COST=COS(TETA)

!coordinate transformation
UD=UPRE*SINT-UPIM*COST
UQ=UPRE*COST+UPIM*SINT

!flux linkages (with and without speed included in the model)
IF (SPE.EQ.1) THEN
  PSIQ=-((UD+ID*RA)/SPEED)
  PSID=-(UQ+IQ*RA)/SPEED
ELSE
  PSIQ=-(UD+ID*RA)
  PSID=(UQ+IQ*RA)
ENDIF

!field winding current
IF=(PSIQ-(PSID+XA*ID)*XAD/XRF)/(1-XAD/XRF)

!negative & zero sequence
G1=UBASE*G2
B1=UBASE*B2
G00=UBASE*G0
B00=UBASE*B0
INRE(I)=(-UNRE(NODE1)*G1+UNIM(NODE1)*B1)
INIM(I)=(-UNIM(NODE1)*G1-UNRE(NODE1)*B1)
I0RE(I)=(-U0RE(NODE1)*G0+U0IM(NODE1)*B00)
I0IM(I)=(-U0IM(NODE1)*G0-U0RE(NODE1)*B00)
IN2=INRE(I)**2+INIM(I)**2

!negative seq. torque
TE2=-(R2-RA)*IN2

!electrical torque
TE=IQ*PSID-1D*PSIQ+TE2

!deviation angle
TETA: (SPEED-1)*W0-.D/DT.TETA=0

!machine speed
SPEED: TM-TE-.D/DT.SPEED*2*H-D*(SPEED -1)=0

!field winding flux linkage
PSIF: UP-IF-.D/DT.PSIF*TF=0

! mmf in the d resp q axis
FWD=PSID+ID*XA
FWQ=PSIQ+IQ*XA

!no saturation
WD=FWD
WQ=FWQ

!currents of d-and q-axis
ID: IF+IDD-ID*XAD-WD=0
IQ: IQ1+I0Q-IQ*XAQ-WQ=0

! damper winding voltage and flux linkage equations for TYPE 1A
PSIDD: 0-IDD+TD*.D/DT.PSIDD
PSIQ1: 0-IQ1+TQ1*.D/DT.PSIQ1
PSIQ2: 0-I0Q+TQ2*.D/DT.PSIQ2
IDD: PSIDD-IDD*(1-XAD/XDD)+XAD/XDD*FWD
TQ1: PSTQ1=IQ1*(1-XAQ/XQ1)+XAQ/XQ1*FWQ
TQ2: PSIQ2-I0Q*(1-XAQ/XQ2)+XAQ/XQ2*FWQ

ENDIF !end if(start)

!positive sequence
VC=SQRT(UPRE**2+UPIM**2)
IPRE(I)=ID*SINT+IQ*COST
IPIM(I)=IQ*SINT-ID*COST
IPRE(NODE1)=IPRE(I)*IBASE
IPIM(NODE1)=IPIM(I)*IBASE

!neg. & zero seq.
IF (.NOT.START) THEN
  INRE(NODE1)=INRE(I)*IBASE
  INIM(NODE1)=INIM(I)*IBASE
  I0RE(NODE1)=I0RE(I)*IBASE
  I0IM(NODE1)=I0IM(I)*IBASE
ENDIF
Appendix D

Block diagram for synchronous machine model Type 1A, by Jonas Persson.
References


Abbreviations

DSL Dynamic Simulation Language used in Simpow when programming e.g. synchronous machine models. Developed by Kjell Anerud

dq0-axis direct-, quadrature- and zero axis following the rotation of the rotor of a machine

dq0-transforme transformation from three-phase stator quantities to rotor quantities

FORTRAN FORmula TRANslation, the first high-level computer language. Developed by Jim Backus

IEEE Institute of Electrical and Electronics Engineers, Inc

KTH Kungliga Tekniska Högskolan, the Royal Institute of Technology

Masta module of Simpow, where instantaneous models are simulated per unit system, unit-less system, often used in power systems analysis

Simpow Power system simulation software, developed by ABB

SI-units Le Système International d’Unités, adopted by the Eleventh General Conference on Weights and Measures, held in Paris in 1960, for a universal, unified, self-consistent system of measurement units based on the MKS (meter-kilogram-second) system

Transta module of Simpow, where fundamental frequency models are simulated

Sub-indices

0 the zero axis of the rotor reference system
0 zero sequence
0 extra index at time zero
1 positive sequence
2 negative sequence
α phase α, of the two-phase global reference system
β phase β, of the two-phase global reference system
ε leakage inductance
a phase a, of the three-phase system
a armature (leakage) resistance, inductance, reactance
b phase b, of the three-phase system
c phase c, of the three-phase system
d the direct axis of the rotor reference system
dq0 the dq0-transformed three-phase stator reference system
netw base per unit bases for the network
node base per unit bases for the machine node
n index for nominal quantities of the synchronous machine
q the quadrature axis of the rotor reference system
R the three-phase rotor reference system
S the three-phase stator reference system
Sbase per unit bases for the synchronous machine system
t generator node quantities

Super-indices

′′ subtransient
Detailed Description of Synchronous Machine Models Used in Simpow

* transient

\* complex conjugate

**Symbols**

\( \delta, \theta \) electrical displacement angle, Transta resp. Masta

\( \mu \) translation or screening factor

\( \sigma \) decrement factor

\( \tau \) time constant

\( \omega \) frequency, rad/s

\( \Delta \omega \) rotor speed deviation

\( \omega_{\text{mech}} \) mechanical speed

\( \Psi, \psi \) flux linkage, SI-units resp p.u.

\( \Re \) reluctance

\( C_I \) current transformation constant

\( C_U \) voltage transformation constant

\( D \) damping factor, p.u.

\( f \) frequency, Hz

\( f \) saturation function

\( H \) per unit inertia constant

\( I, i \) current, SI-units resp p.u.

\( J \) moment of inertia, SI-units

\( L, l \) inductance, SI-units resp p.u.

\( M, m \) torque, SI-units resp p.u.

\( P, p \) active power, SI-units resp p.u.

\( Q, q \) reactive power, SI-units resp p.u.

\( R, r \) resistance, SI-units resp p.u.

\( S \) imaginary power, SI-units

\( U, u \) voltage, SI-units resp p.u.

\( W \) magneto motive force

\( X, x \) reactance, SI-units resp p.u.

\( Z, z \) impedance, SI-units resp p.u.