Power Generation Scheduling
A free market based procedure with reserve constraints included

by

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Preface

This thesis is submitted in partial fulfillment of the requirements for the degree of Doktor Ingeniør at the Department of Electrical Power Engineering at the Norwegian University of Science and Technology (NTNU).

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I would like to thank my supervisors Professor Hans H. Faanes and Professor Arne Johannesen. Professor Hans H. Faanes has been my principal supervisor.

I would also like to thank colleagues at SINTEF Energy Research for useful discussions about the topic.

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Einar Ståle Huse
Summary

This thesis deals with the short-term scheduling of electric power generation in a competitive market. This involves determination of start-ups and shut-downs, and production levels of all units in all hours of the optimization period, considering unit characteristics and system restrictions. The unit characteristics and restrictions that are handled are: Minimum and maximum production levels, fuel cost function, start-up costs, minimum up time and minimum down time. The system restrictions handled are power balance (supply equal to demand in all hours) and spinning reserve requirement.

This thesis has two main contributions:

1. A new organization of an hourly electric power market, that simultaneously sets the price of both energy and reserve power is proposed. A power exchange is used as a trading place for electricity. The responsibilities of the power exchange is to balance supply and demand bids, and to secure enough spinning reserve. Routines for bidding and market clearing are developed.

2. A computer program that simulates the proposed electricity market has been implemented. This program can also be used as a new method for solving the single owner generation scheduling problem. Simulations show excellent performance; both low computation time and good production schedules (i.e. low production cost).

The main simplifications that are applied:

1. Network restrictions neglected
2. Bid calculation by the units is based on a price forecast for the near future (24-168 hours). The units use only the expected value of future prices and treat them as if they were deterministic
3. Emphasis has been set on the thermal units

With the proposed market, all optimization is done locally, by each generator. This means that each optimization problem is small and can easily be solved. The tasks of the power exchange are well defined, and involves no optimization. It is just the simple matching of supply and demand bids.

A computer program simulating this market is developed, and used on several test cases. This program, called SimCom (Simulated Competition), can also be used to solve the single owner scheduling problem. and the solutions by this program are very good (i.e.
low schedule cost). Results from three different test cases are reported. The cases vary in number of units (9 to 110 units) and optimization period (24 and 168 hours). SimCom has shown excellent performance, schedule cost has been near-optimal, and very good compared to other optimization techniques. Computation time is at the same level as Lagrangian Relaxation.

Simulations show that it is possible to reach efficient schedules through the proposed electricity market.
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Abbreviations
DP Dynamic Programming
LR Lagrangian Relaxation
MDT Minimum down time
MUT Minimum up time
NOK Norwegian Kroner (monetary unit)
PX Power Exchange
SAC Short-run Average Cost
SAVC Short-run Average Variable Cost
SMC Short-run Marginal Cost

Symbols
λ Price of energy (NOK/MWh or $/MWh)
μ Price of reserve power (NOK/MW or $/MW)
p Production level (in MW)

In equations, upper case letters is used for constants and lower case for variables.

Nomenclature
Ancillary services Services that the System Operator may develop, in cooperation with market participants, to ensure reliability and to support the transmission of energy from generation sites to customer loads. Such services may include: regulation, spinning reserve, non-spinning reserve, replacement reserve, voltage support, and black start

Deregulation The elimination of regulation from a previously regulated industry or sector of an industry. In the electricity industry the production and supply can be deregulated (prices no longer regulated),
whereas the natural monopolies transmission and distribution are considered natural monopolies

**Dispatch**

The decision of how much each of the committed units will produce

**Distribution**

Distribution involves the transfer of electricity from the supply points of transmitters (see Transmission) to consumers

**Natural Monopoly**

A situation where one firm can produce a given level of output at a lower total cost than can any combination of multiple firms. Natural monopolies occur in industries which exhibit decreasing average long-run costs due to size (economies of scale). According to economic theory, a public monopoly governed by regulation is justified when an industry exhibits natural monopoly characteristics

**Reregulation**

The design and implementation of regulatory practices to be applied to the remaining regulated entities after restructuring of the vertically-integrated electric utility. The remaining regulated entities would be those that continue to exhibit characteristics of a natural monopoly, where imperfections in the market prevent the realization of more competitive results and where, in light of other policy considerations, competitive results are unsatisfactory in one or more respects. Reregulation could employ the same or different regulatory practices as those used before restructuring.

**Retail sales**

The sale of electricity to small end-users (households etc.)

**Scheduling**

The decision of which units to operate, and their power output, during a certain time period (typically from one day to one week). Scheduling equals unit commitment plus dispatch

**Spinning Reserve**

Unused capacity available from units connected to and synchronized with the grid to serve additional demand

**System Operator**

An operator responsible for maintaining instantaneous balance of the grid system. The system operator performs its function by controlling the dispatch of flexible plants to ensure that loads match resources available to the system

**Transmission**

Transmission involves the transfer of electricity from generators over the main grid to supply points (from which distribution companies transfer the electricity to the end-users)
Unit commitment  The decision of which units to operate during a certain time period

Water value  Water value or incremental worth of water is the expected future worth of water stored in the reservoir

References
(x.y)  Reference to equation y in chapter x
[x]  Literature reference
Chapter 1
Introduction

This chapter describes the background and the objectives for this work. It also points out the limitations/simplifications that are applied. Finally it describes the organization of this thesis.

1.1 Background

Deregulation and market competition is the new paradigm in the electricity sector. In Norway the electricity sector was deregulated in 1991, but a spot market for power exchange between generation companies had existed since 1971. Similar systems are implemented or discussed in several countries around the world. Market systems (i.e. bidding and market clearing procedures) that ensure efficient utilization of resources are therefore of interest.

Concepts and solutions for coordinating production and extension of electric power systems exists today for hydro dominated systems, for thermal dominated systems and to a certain extent also for the mixed hydro-thermal system in a monopolistic environment (the single-owner-problem with central planning).

For the general hydro-thermal power system in a competitive market there is no “complete solution”.

1.2 Objectives and scope of work

The goal is to create a system for hydro-thermal scheduling in a free market framework, where all optimization is done locally. At present a simple supply-demand clearing procedure is used in the common electricity market of Norway and Sweden [9], and spinning reserve requirements are not handled within the market clearing process.
One of the main guidelines when designing the suggested electricity market has been simplicity. The reason for simplicity is that the participants have to understand the market rules for the market to function properly. The more complex the market rules are, the more likely it is that it will not operate as intended.

1.3 Limitations

The work is limited to the short term scheduling of power plants, i.e. a planning horizon of 24 to 168 hours. The object of short term planning is to create schedules for when to commit and decommit units, and their production levels. Inflows to hydro reservoirs and loads are assumed known in this time frame.

This work is restricted to the case when all power stations are feeding into a concentrated network represented by a single bus bar. Transmission losses and transmission limits are neglected.

Since the combined Nordic market can be seen as a well functioning electricity market for a hydro dominated electricity production system, emphasis has been set on the thermal problem. Another reason for focusing on thermal units is that the spinning reserve requirement and the unit commitment problem (due to high start-up costs) are more important in a thermal system. Bidding procedures for hydro units are only briefly discussed.

1.4 Organization of the thesis

Chapter 2 presents the problem definition. Existing theory and solutions are described in chapter 3. In chapter 4, a new electricity market is proposed with procedures for market clearing and bidding. This new market includes the spinning reserve requirement in the market clearing process.

Chapter 5 contains a description of a computer program which simulates the proposed market.

Results from simulations of the proposed market are shown in chapter 6. The results are discussed in chapter 7, and topics for future research are identified. Chapter 8 contains the final conclusions of the thesis.

Chapters 4 and 5 contains the original contribution of this thesis. Chapters 4 presents the
proposed electricity market, and chapter 5 presents a computer program which simulates the proposed market. This computer program can also be used to solve the traditional single owner scheduling problem.
1.4 Organization of the thesis
Chapter 2

Problem definition, short-term scheduling

This chapter contains a description of the short term generation scheduling problem. It starts with a general overview, and ends with the specific cost-functions used later in this thesis.

2.1 Introduction

The goal of this work is to design an short-term electricity market where all optimization is done locally by each generating company (and each consumer). The work is limited to the short term scheduling of the electricity production with time horizon from a day to a week (24 to 168 hours). Long term impacts are not considered.

Short-term scheduling (1 day to 1 week) involves the hour-by-hour scheduling of all generation in a system to achieve maximum social surplus (with firm load minimization of production costs gives the same result). In such a scheduling problem, the load, hydraulic inflows, and unit availabilities are assumed known.

Power transmission and distribution clearly exhibit economies of scale, and is therefore considered as natural monopolies. As monopolies, transmission and distribution must be regulated by a regulatory authority. Network restrictions, tariffs, etc. is not considered in this thesis.

Power production and supply has considerably smaller economies of scale, and is an area where competition is possible.
2.2 Goal

The goal is to design an electricity market that ensures efficient utilization of the electricity system. This means maximization of social surplus (which is the sum of consumer and producer surplus), while taking care of the unit and system constraints.

The work is limited to the short-term scheduling of power plants. The unit schedules shall be found through market mechanisms (by matching supply and demand bids). All optimization should be done locally by each unit (in the bid calculation).

Long term effects are not considered.

2.3 Global constraints

2.3.1 Power balance

In a power system, power balance must be maintained, i.e. load demand must be balanced by generation supply at all times.

The load forecast for a few days ahead is an important basis of the short term scheduling. In order not to increase the complexity of the problem, most scheduling algorithms represent the forecasted load in the form of stepped curves; the load within a time step is assumed to be constant.

The demand can be firm or price dependent.

2.3.2 Spinning reserve requirements

In power system operation, in addition to the requirement that power must be balanced for the present situation, power balance must be maintained to guarantee continuity of supply even after a disturbance in the system such as the failure of a generator or the outage of a transmission line. To ensure reliability, prudent measures must be taken from both the generation side and the transmission side. More generation capacity than necessary for the forecasted load is scheduled. This gives a readily available reserve to cover unforeseen events if generation failure or sudden demand surge. [37]

2.3.3 Transmission/distribution network influences

In this work, a one bus bar model is used, i.e. all units (both load and generation) are con-
nected to a single bus bar through a line with no losses. Power transfer capacity limits and network losses are ignored. Network tariffing is also ignored.

2.4 Unit constraints and cost functions

Each generator has a set of constraints that must not be violated. The constraints considered in this work are:

- Unit rated minimum and maximum production levels
- Unit minimum up/down time
- Initial condition (whether the unit is running or not, and for how long)

The economics of each unit is described by two cost functions:

- Fuel costs as a function of the production level
- Start-up costs that are dependent on how long the unit has been down

2.5 Mathematical formulation

The generation scheduling problem involves the determination of the start-up and shut-down times as well as the power output levels of all the generating units at each time step, over a specified scheduling period \( T \).

The fuel cost for unit \( i \), \( FC_i \), in any given time interval is a function of the generator power output.

Hydro units is in short term planning normally represented either by water values\(^1\) or by weekly drawdown quantities set by the long term planning. With a predetermined water value, hydro units can be described by a fuel cost function.

The generator start-up cost, \( SC_i \), depends on the time the unit has been off prior to start up. Shut-down costs can be included in the start-up cost.

The total production cost, \( F_T \) for the scheduling period is the sum of the running costs and start up costs for all units:

\(^1\) The water value (or incremental worth of water) is the expected profit from an amount of water not currently used for power production, but left in storage. If the storage is full, the incremental worth of water is zero, since the water will be lost if it is not used for power production.
In general, the overall objective is to maximize social surplus, subject to a number of constraints (see below). In the simulations in this thesis, load is firm. With firm load minimizing total production cost will maximize social surplus. Production costs are therefore reported from the simulations.

Constraints:

- System hourly power balance. Total power generation must equal the load demand, \( P_D \), in all hours

\[
\sum_{i=1}^{N} P_{i,t} = P_{D,t} \quad t = 1, 2, \ldots, T
\]

- Hourly spinning reserve requirements \( R_t \) must be met in each hour (\( u_{it} = 1 \) if unit \( i \) is running in hour \( t \), otherwise \( u_{it} = 0 \)). In this formula, reserve is set equal to the maximum production of the unit minus actual production for units that are running. This is the way reserve is treated in the simulations later in this thesis.

\[
\sum_{i=1}^{N} \left( P_{i,t}^{\text{max}} u_{i,t} - P_{i,t} \right) u_{i,t} \geq R_t \quad t = 1, 2, \ldots, T
\]

- Unit rated minimum and maximum capacities must not be violated

\[
P_{i,t}^{\text{min}} u_{i,t} \leq P_{i,t} \leq P_{i,t}^{\text{max}} u_{i,t} \quad i = 1, 2, \ldots, N, t = 1, 2, \ldots, T
\]

- The initial unit states at the start of the scheduling period must be taken into account

- Minimum up/down time limits of units (MUT/MDT) must not be violated

2.5.1 Cost functions used in this thesis

In this work, a quadratic function is used to represent the fuel costs (this is a frequently
used cost function for thermal units):

\[ FC_i = a_i + b_i p_i + c_i p_i^2 \]  \hspace{1cm} (2.5)

\(a_i, b_i, c_i\) represent unit cost coefficients, while \(p_i\) is the unit power output.

The start up cost in any given time interval is represented by an exponential cost curve:

\[ SC_i = \sigma_i + \delta_i \left( 1 - exp \left( \frac{-T_{off,i}}{\tau_i} \right) \right) \]  \hspace{1cm} (2.6)

\(\sigma_i\) is the hot start-up cost, \(\delta_i\) the cold start-up cost, \(\tau_i\) the unit cooling time constant and \(T_{off,i}\) is the time the unit has been off.

With these cost functions and firm load, the problem definition is the same as in [24], except that shut-down costs are added to the start-up costs.

Modeling of the generation scheduling problem are described in [1] and [6].
2.5 Mathematical formulation
Chapter 3
Market solutions

This chapter contains a short description of existing free market solutions in the electricity industry. This includes the Nordic electricity market and the markets in UK, Argentina and California.

3.1 Introduction

Several countries in the world have deregulated, or are in the process of deregulating their electricity supply industry. In each of these countries, the spot markets are based on one out two models: The centralized scheduling model or the bilateral trade model. The UK electricity market is based on the first, and the Nordic market is based on the latter. These two models are briefly described here. In addition, an extension of the bilateral model proposed in [37] called the Coordinated Multilateral Trade model is described.

3.1.1 The centralized scheduling model

In this model, all utilities combine to form a "super-utility" (a centralized pool), and the market structure is altered to suit this super-utility. The centralized pool keeps the traditional responsibilities, such as ensuring instant power balance, maintaining network reliability and security, and coordinating transmission access and services. Every trade is now essentially required to be with the pool. The pool determines which trades to accept and execute and which trades to reject so that the system is safe, and sets the price at which trades are settled so as to promote economic efficiency (marginal cost pricing).

With its dictatorial power, the pool can, in principle, enforce any of a large number of operating points. Advocates refer to this obvious point when they assert that the pool can operate efficiently. However, the pool has no incentive to operate efficiently, and need to be regulated.

Reference: [37]
3.1.2 The Bilateral model

The bilateral model is based on the principle that free market competition is a route to economic efficiency. In this model suppliers and consumers independently arrange trades, setting by themselves the amount of generation and consumption and the corresponding financial terms, with ideally no involvement or interference by the power system operator. Economic incentives will lead generators to find the best-paying customers and consumers to find the cheapest generators. So long as consumers or generators do not have significant market power, these trades will lead to short term economic efficiency.

A voluntary power exchange (as in the Nordic power market) does not conflict with the bilateral model, as it is just another opportunity for the market participants to meet and trade electricity.

The bilateral model faces a fundamental problem which detract from its ability to promote free market competition. The lack of coordination among the independent trades can lead to a violation of transmission network constraints. The network constraints arise from loading limits on transmission equipment and from the requirement that the network must be operated in a secure state.

Reference: [37]

3.1.3 The Coordinated Multilateral Trade model

The Coordinated Multilateral Trade model is a proposal of a new operating paradigm in which the decision mechanism regarding economics and reliability (security) of system operation are separated. Economic decisions are carried out by private multilateral trades among generators and consumers. The function of reliability is coordinated through the system operator who provides publicly accessible data (load and loss vectors), based on which generators and consumers can determine profitable trades that meet the secure transmission loading limits. Each trade accounts for its share of the losses. Efficiency is attained through the invisible hand of the market.

A broker arranges trades and bears responsibility for arranging generation to compensate

---

1. The load vector is used to calculate the net influence of a trade on a congested line. If the net influence is that the load on that line is reduced, the trade is feasible. The loss vector is used to calculate the trade’s share of the network losses.
for losses and to insure that the trading is feasible. The broker can be any of the contracting parties, or an independent third party. All trading is by bilateral or multilateral contracts. The authors prove that with network restrictions included, multilateral contracts are needed to reach the optimal solution. Situations can occur where there are no more profitable bilateral trades, but where profitable multilateral trades (with three or more participants in the trade) exist.

The power system operator evaluates if a trade is feasible or not. Unfeasible trades are curtailed. The power system operator has no information about the economic aspects of the trades. The responsibility of the power system operator is to curtail unfeasible trades, and to calculate and distribute the load and loss vectors.

Trading model:

- Initial trading until a line becomes congested. Then the power system operator curtails trades to reduce flow on the congested line to its transfer limit.
- The power system operator calculates the Loading vector and broadcast this set of numbers to everyone (Data required for calculating the Loading vector is basically what is required for a load flow study, i.e. transmission network data and power generation and consumption data for the trades).
- Based on the loading vector, the participants can arrange new trades that do not increase the loading on the congested line. It is possible that a profitable trade between participants can result in overload of another transmission limit besides the original congestion. Then the power system operator must curtail this trade to ensure that the transfer limit is not exceeded, and then broadcast the Loading vector corresponding to the additional congestion. In general, if several of transmission transfers are congested, the power system operator can produce Loading vectors, one for each transfer limit and participants can use these vectors to ensure that their trade does not overload any of them.
- The sequence of coordinated multilateral trades continues until there is no more profit to be made and an optimal solution is reached.

For a detailed description of the Coordinated Multilateral Trade model, see [37].

2. Adam Smith’s invisible hand hypothesis: In a competitive market, a powerful "invisible hand" assures that resources will find their way to where it is most valued, thereby enhancing the "wealth" of a nation
3.2 Norway, Sweden and Finland

The electric power production in Norway in almost entirely based on hydro power. In 1997, 99.41% of the production was from hydro plants, 0.59% from fossil fired units and 0.01% from wind power. Total power production was 112 TWh.

In Sweden most of the electricity comes from hydro and nuclear units. In 1997, 46.17% was from nuclear plants, 47.11% hydro, 6.58% from thermal other than nuclear and 0.14% from wind power. Total production was 145 TWh.

In Finland most of the electricity is generated by thermal units. In 1997, 30.41% was from nuclear plants, 51.57% other thermal, 18.00% hydro and 0.03% wind power. Total production was 66 TWh.

The Norwegian and Swedish electricity systems are interconnected by several AC connections with a total capacity of ca. 3,500 MW. The exchange capacity between Sweden and Finland is ca. 1,300 MW, and between Finland and Norway 70 MW (all AC-connections).
The largest power producers are:
- Vattenfall (Sweden), ca. 22% of installed capacity
- Sydkraft (Sweden), ca. 11% of installed capacity
- Imatran Voima Oy (Finland), ca. 11% of installed capacity
- Statkraft (Norway), ca. 10% of installed capacity

There are 230 distribution companies in Norway, 270 in Sweden and 75 in Finland. Some of these distribution companies also own generation.

The national transmission grid is owned by Statnett in Norway, by Svenska Kraftnät in Sweden and by Suomen Kantaverkko (Fingrid) in Finland.

### 3.2.1 The Nordic electricity market

This information is mainly extracted from [4].

Deregulation and market competition was introduced in Norway with the Energy Act of June 1990 which was effective from January 1, 1991. Sweden joined the existing Norwegian market structure January 1, 1996. Finland introduced competition with its Electricity Market Act of June 1 1995, and joined the Nordic spot market June 15 1998.

In contrast to deregulated systems in England and elsewhere, the Nordic system does not include a central scheduling/dispatching entity. Scheduling is the responsibility of the individual generating companies with a power exchange (Nord Pool) and system operator (Statnett in Norway, Svenska Kraftnät in Sweden and Suomen Kantaverkko in Finland) being responsible for market clearing and system coordination respectively. [9]

Regulatory authorities for the transmission systems are NVE in Norway, Energimyndigheten in Sweden and Sähkömarkkinakeskus in Finland. Entities owning both generation and distribution are required to account for them separately in Norway and Finland, and to separate operations organizationally in Sweden.

Nord Pool operates two markets, a futures market and a spot market. The national system operators each operate their own regulating market for real-time operation (called the balance service in Sweden). The futures market permits purchase of weekly base or peak load contracts for up to three years in advance to manage price risk.

Nord Pool is a neutral and independent exchange for electric power where all partici-
pants who wants to, can buy and sell electric power in a flexible and simple way. It is optional to participate in the power exchange.

In addition to the organized markets it is possible to make bilateral contacts.

**The spot market**

The spot market accept bids for all 24 hours of each day until noon (12.00) of the preceding day. Bids are in the form of linear segment price versus quantity curves for both generators and loads. Bids are accepted by FAX, or electronically using a communications package called EDIEL based on UN/EDIFACT, and X.400 e-mail. The market is settled for each day by 15.00 on the preceding day. The bids are aggregated into separate price versus quantity curves for supply and demand. These curves are crossed to obtain a system price. Spot market participation is not mandatory. About 16 percent of energy in the Nord Pool market area is traded on the spot market.

Main features of the spot market:

- Each day is divided into 24 hours with a price for each hour
- The spot market is active every day
- Participants are required to make balance by buying and selling in the spot market
- The price that creates balance (supply equals demand), disregarding network flow restrictions, is defined as the system price
- Turnover in Nord Pools spot market in 1997: 43.6 TWh

**The futures market**

The futures market is a contract market for hedging or trading. Main features:

- Trading in week contracts, blocks (4 weeks) and seasons
- Hedging on future buying or selling
- Trading on price expectations between the participants
- Risk levelling between the participants
- Continuous trading every week day between 12.30 PM and 15.00 PM
- Daily calculation of profit and loss against daily price changes, and calculation of margin
- Daily clearing against the system price in the delivery week (financial clearing)
- Turnover in Nord Pools futures market in 1997: 53.6 TWh
The regulating power market

Unpredicted and unplanned deviations between generation and consumption is balanced by the national system operators. The price is set by a market based price list. The regulating object with the most favorable price is chosen first and so on in a up/down regulation situation. It is exclusively used for real removal of deviations. It is a requirement to have physical ability to regulate production to take part in this market.

The regulating market accepts bids from participants to raise or lower energy generation from scheduled values. Bids are accepted for each day between 15.00 and 19.30 on the preceding day. Participants must be able to respond within 15 minutes in Norway, and within 10 minutes in Sweden. When the system operator decides that regulation is necessary, it buys the cheapest block of regulating power. Dispatch is by telephone. At the end of the hour, the system operator decides the price for purchased regulation at the price of the most expensive purchased block. All network users are charged for regulation based on deviation from scheduled hourly energy values.

The network regime

Each generator and load pays point tariffs to the network to which it is connected. There are three network levels, national, regional and local. Each network pays a point tariff to the higher level network to which it is connected. User point tariffs give the user access to all network levels for buying or selling energy. Thus, a load attached to a local network, paying the local network point tariff, can purchase energy from a generator attached to the national grid.

The point tariff has three components. The investment charge is a one time charge imposed for major new connections. The energy charge, per MWh, is based on incremental loss coefficients (calculated in advance, bimonthly). The capacity charge, based on peak MW consumed or generation capacity (physical ability in Norway, declared capacity limit in Sweden), compensates the networks for their remaining expenses.

Congestion management

Sweden and Norway have different philosophies of congestion management. The existing system permits these philosophies to coexist without conflict. Norway seeks to effectively prevent congestion by using the spot market settlement process. When congestion is predicted, the system operator declares that the system is split into price areas at pre-
dicted congestion bottlenecks. Spot market bidders must submit separate bids for each price area in which they have generation or load. If no congestion occurs during market settlement, the market will settle at one price, which will be the same as if no price areas existed. If congestion does occur, price areas are separately settled at prices that satisfy transmission constraints. Areas with excess generation will have lower prices, and areas with excess load will have higher prices. Market income from price difference goes to the system operator, and is used to reduce the capacity fee. Bilateral contracts that span price areas must purchase the load’s energy in its price area, in order to account for the contribution to congestion, and to expose the contract to the financial consequences of congestion. It is the only instance of mandatory spot market participation.

Sweden’s philosophy is that the transmission system should not affect the market solution. Consequently, Sweden is always one price area. However, Sweden varies the capacity charge portion of its point tariff based on geography. Power flow in Sweden is always from north to south, so generation is charged more, and load less, in the northern part of the country. This affects generation costs, and thus the bids made to the market, deterring some congestion. Congestion in post-market schedules or appearing in real time is corrected by purchase of generation raise and lower energy blocks from the system operator regulating markets. This is known as buyback.

**Retail sales**

All loads in Norway, Finland and Sweden, including individual residential loads, are legally entitled to a free choice of energy supplier. Hourly energy consumption (MWh) values are necessary for accounting among energy suppliers. In Sweden, consumers choosing a supplier other than their directly connected distribution network must install hourly energy metering to provide settlement values. In Norway and Finland, each network is responsible for supplying hourly MWh values for loads connected to it, to the suppliers of those loads. Loads without hourly metering are assigned a load profile based on the load profile of all unmetered loads within their connected network, and have the option to purchase hourly energy meters. No fee is charged by the network for changing energy suppliers.

SINTEF Energy Research has made a report about the implementation and experiences of the Nordic power market [20].
3.3 United Kingdom

This is based on information contained in [5], [7], [8], [32], [33], [34], [35] and [36].

The statistics are available at the web pages of the Electricity Association\(^1\).

3.3.1 The UK electricity system

From an organizational viewpoint, there are three separate electricity systems in the United Kingdom: England and Wales, Scotland, and Northern Ireland. The approximate size of each market can be seen from the respective peak demands in 1995/96: England and Wales 48,811 MW, Scotland 5,849 MW and Northern Ireland 1,515 MW. The English and Scottish systems are interconnected with 1600 MW transmission capacity, and there is also a 2,000 MW direct current link between England and France.

The largest power production companies are:

- National Power (fossil-fired), 31.38% market share in 1995/1996 fiscal year\(^2\)
- PowerGen (fossil-fired), 23.1% market share in 1995/1996 fiscal year
- Nuclear Electric (nuclear), 22.49% market share in 1995/1996 fiscal year

The distribution system is divided into twelve regional electricity supply companies (RECs), which are regulated monopolies.

The National Grid Company (NGC) provides transmission services from generators to the RECs, coordinates transmission and dispatch of electricity generators and runs the electricity spot market.

---

2. Fiscal years runs from April 1 to March 31 of the following year
3.3.2 The electricity market in England and Wales

The Electricity Act of 1989 introduced a competitive structure (effective from April 1, 1990), the completion of which is due to occur in 1998, when the entire supply market is opened for competition. This market does not include Northern Ireland where a separate regulatory regime exists. Scotland and France operates in this market as External Pool Members (EPM).

In 1990 and 1991, the electricity companies in Great Britain were privatized, except for the nuclear plants which are now owned by Nuclear Electric, a publicly owned company. The competitive market was opened to customers of 1 MW and above. In April 1994 the market opened to customers in the 100 kW-1 MW market. During 1998 the supply of electricity to homes and small businesses is planned to be opened to full competition.

The restructuring has transformed the electricity supply industry into four separate sub-industries:

1. Generation - the production of electricity
2. Transmission - the transfer of electricity in bulk across the country
3. Distribution - the delivery of electricity over local networks

4. Retail sales - the acquisition of electricity and its sale to customers

Both transmission and distribution are identified as natural monopolies and prices are thus regulated. Incentive regulation for these monopoly elements of the system is based on the "RPI-X" formula used in utilities, including gas and telecoms, privatized previously. Under this formula, prices of the monopoly elements of the consumer price are allowed to increase by the general rate of inflation as measured by the Retail Price Index (RPI) minus a term, "X", which the utility must recover by increasing its internal efficiency. The level of "X" is set for 3-5 years forward to allow a stable and predictable financial framework for the utility and its customers.

Competition is introduced in generation and retail sales. RECs are required to allow competitors to transfer electricity over their distribution systems at the same price they charge to themselves to provide this service to their retail customers located in their own service area. All RECs have an obligation to provide electricity to every customer in their area. All competing supply companies have an obligation to publish their terms and supply a customer on request.

NGC runs both the financial and physical side of the UK electricity market. NGC determines both half-hourly market clearing prices and it runs the physical national electricity grid, making generator dispatch decisions in real-time to manage congestion on the grid and provide the ancillary services necessary to guarantee reliable power to all final customers.

The spot market is mandatory, and does not allow physical bilateral trades between generators and their customers. Unless a generating facility is dispatched by NGC as part of the day-ahead spot market clearing process, that plant cannot produce electricity. A plant that is dispatched by NGC will receive the market clearing price for all MWhs it produce during that half-hour.

**Bidding** (from [32])

No later than 10:00 a.m. each day, all operators of generating stations subject to central
Despatch, i.e. of 100 MW DNC\(^1\) or over, will inform NGC of:

- the "offer prices". These are the prices at which each station’s operator is willing to operate, at different levels of output, each separate available generating unit for each half hour the next day. The offer price will consist of a start up price, a fixed price and up to three "incremental prices" per unit of electricity produced
- the declared availability of their plant for each half-hour of the next day
- the prices at which they are willing to keep each unit in a standby mode
- the state of readiness of the unit, i.e. whether in operation, on standby or shut down
- the prices at which they are willing to operate the units for a limited period at higher levels of output than the declared availability

There are no demand side bids. Demand is forecasted by NGC.

Based on the bids and the forecasted demand, NGC sets the production schedule and determines the settlement prices.

**Derivation of Pool Prices** (from [32])

1. NGC will derive an operating regime for all the plants on the basis of the forecast demand and declared availabilities of plants but assuming that the transmission constraints did not exist. The regime derived in this way is known as the "unconstrained schedule" and is distinct from the schedule used to despatch plants.

2. Each generating unit’s offer prices will be converted into a "Generator Price" which is the average price (per kWh) of providing power at the unit’s maximum declared available output, excluding the start-up charge

3. In "Table A"\(^2\) periods, SMP (System Marginal Price) will be defined as the highest Generator Price of plant required to operate in each half hour according to the unconstrained schedule, as long as the plant does not receive a "marginal plant adjustment". In "Table B" periods, SMP will be the highest incremental price of a plant that is not labelled "inflexible".

4. The capacity element is LOLP(VOLL-SMP)

---

1. Declared Net Capability (DNC): The maximum power available for export on a continuous basis minus any power imported by the station from the network to run its own plant.
2. There will be times, usually of low demand, when the aggregate offered capacity of the scheduled sets exceeds demand by a predetermined margin. These times are known as Table B periods; all other periods are Table A periods.
LOLP is the "Loss of Load Probability". This is the probability of capacity being inadequate to supply demand in the particular half hour because of a sudden unexpected increase in demand or a sudden failure of plant such as a generating station. It will be calculated by NGC; and 

VOLL is the "Value of Lost Load". It is a measure of the price that pool customers are willing to pay to avoid a loss of supply. It will be set at a level to ensure that the quality of supply will be maintained (see [32], page 25)

5. The prices paid every half hour to generators for each kWh produced, the "pool input price", pip, will be:

\[
pip = \text{SMP} + \text{capacity element} = \text{SMP} + \text{LOLP} \times (\text{VOLL} - \text{SMP})
\]

6. Generators will also be paid for reserve, marginal plant operation, and for any ancillary services. In addition, they will receive payments to recompense them for transmission constraints and for forecasting errors and for having the plant available to operate. Generators will be penalized if they do not follow NGC’s instructions.

7. The costs associated with 6. and the transmission constraints will be spread over the units of electricity purchased through the pool during "Table A" periods. This results in an "uplift" being added to pip to arrive at "pool output price" (pop). The difference between pip and pop covers the costs of:

- reserve
- availability of plant
- forecasting errors
- transmission constraints
- ancillary services
- marginal plant adjustment

3.4 The Californian electricity system

The total consumption of electricity in California is ca. 250 TWh. Electricity is generated from natural gas, oil, nuclear, hydro, and geothermal resources. Electricity is also imported into the state from neighboring states, Canada and Mexico.

The largest utilities in California are:

- Pacific Gas & Electric (PG&E)
- Southern California Edison (SCE)
- San Diego Gas & Electric (SDG&E)
3.4 The Californian electricity system

These three utilities provide almost 70 percent of the electricity in California.

### 3.4.1 The electricity market in California

The deregulation process of the US electricity industry was initiated by the 1978 passage of the Public Utility Reform Policy Act (PURPA). The Energy Policy Act of 1992 was a further step towards federal deregulation. Many states have initiated their own deregulatory efforts parallel to the federal initiatives. The 1996 signing of Assembly Bill AB 1890 put forth deregulation in California, establishing an Independent System Operator and a Power Exchange, to start operations on January 1, 1998. Market operations started March 31, 1998, after a delay of three months. The new deregulated market structure is to be fully implemented by March 31, 2002, after a transition period of 4 years.

**California Power Exchange (PX)**

- The California Power Exchange is a non-profit corporation; its primary purpose is to provide an efficient, competitive energy auction that meets the loads of PX customers at market prices
- The PX is open on a nondiscriminatory basis to all suppliers and purchasers
- The Power Exchange’s rules and services are regulated by the Federal Energy Regulatory Commission (FERC)
- PG&E, SCE and SDG&E must buy and sell electricity through the PX for the first four years
- The PX determines the price of electricity on an hourly basis for the Day-Ahead and Hour-Ahead markets, by matching the demand and supply bids submitted by PX participants

**California Independent System Operator (ISO)**

- Although PG&E, SCE and SDG&E continue to own their electric transmission facilities, operational control of these facilities is turned over to the ISO
- The ISO’s rules and service charges are regulated by the FERC
- The ISO will ensure that all electricity buyers and sellers have an opportunity to use the transmission system
- The ISO procures ancillary services and performs real-time balancing of load and generation

1. Information about the PX is available at their web-pages at [http://www.calpx.com](http://www.calpx.com)
2. Information about the ISO is available at their web-pages at [http://www.caiso.com](http://www.caiso.com)
Day-Ahead Market

For each hour of the 24-hour scheduling day (starts and ends at midnight):
1. Sellers bid a schedule of supply at various prices (price-quantity bid). Buyers bid a schedule of demand at various prices.
2. The price is determined by matching supply and demand bids.
3. Then sellers specify the resources that will produce the power sold, and buyers specify the delivery points for the power purchased.
4. PX schedules supply and demand with the ISO for delivery.
5. Supply and demand are adjusted to account for congestion and ancillary services.
6. The PX finalizes schedules.

Hour-Ahead Market

The Hour-Ahead market provides a means for participants to buy and sell so as to adjust their day-ahead commitments based on information closer to the transaction hour. It is similar to Day-Ahead, except:
- Trades are for 1 hour (bid 2 hours ahead).
- Available transmission capacity is reduced by Day-Ahead trades.

In addition to these organized markets for wholesale power trade, competition is also introduced in retail sales. Retail customers are free to choose their electricity provider. The distribution companies are not allowed to charge switching fee from customers choosing a supplier other than the local one.

References: [2] and [3].

3.5 The electricity industry in South America

Several countries in the South America has deregulated their electricity industry, with Chile as the pioneer. Chile started its deregulation process in 1978 with a new electricity law promulgated in 1982. Argentina followed with an aggressive process started in 1991, a new law approved in 1992 and a privatization process that is still going on. Several South American countries followed, with Peru (1993), Bolivia (1994) and Colombia (1994) promulgating deregulating laws in line with the Argentinean and Chilean initiatives. Brazil and Ecuador are undergoing similar processes. [26]

Here follows a brief description of the electricity market in Argentina.
3.5.1 The Argentinian electricity law

Argentina is a country with a population of 33 millions; it has two interconnected electricity systems, the main one delivering 10,000 MW and 60,000 GWh in 1994 through a 500, 220 and 132 kV network. With a present total installed capacity of about 18,000 MW, 42% of the annual production is hydro, 43% thermal by natural gas and 15% nuclear. After privatization, supply has been diverted to 39 generation companies (26 thermal + 13 hydro). There are 5 transmission companies (one for the high voltage network and 4 for the regional grids) and 25 distribution companies (of which 6 are private, the remaining 19 still owned by the Provinces (States)). [26]

The ‘Argentine Electricity Act’ of January 1992 divides the electricity industry into three sectors: generation, transmission and distribution. These sectors are vertically dis-integrated, and the controlling stake of a generation company, distribution company or a large user can not control a transmission company. Generation companies are restricted from holding any more than 10% of the market. Transmission companies and distribution companies require licences to operate. Hydroelectric plants require a licence for exploitation of natural resources, thermal plants do not require a licence.

The generation sector is organized on a competitive basis with independent generation companies selling their production on the Wholesale Electricity Market (WEM) or by private contracts with certain other market participants (bilateral contracts).

Transmission is organized on a regulated basis. Transmission companies are required to provide third parties access to the transmission systems they own and are authorized to collect a toll for transmission services. Transmission companies are prohibited from generating or distributing electricity. The major transmission company is Compañía de Transporte de Energía Eléctrica en Alta Tensión S.A.

Distribution involves the transfer of electricity from the supply points of transmitters to consumers. Distribution companies operate as geographic monopolies, providing service to almost all consumers within their specific region. Accordingly, distribution companies are regulated as to rates and are subject to service specifications. Distribution companies may acquire the electricity needed to meet consumer demand on the WEM (by seasonal contracts) or from contracts with generation companies.

Large users (consumers of more than 1 MW of capacity) are free to choose eligible sup-
plier, and are also allowed to buy at the WEM spot prices.

Generation scheduling is performed by the Argentinean pool, CAMMESA, without regard to the contracts among generation companies and distribution companies or large users. All generation companies declare their variable costs of each generator, including the energy price (water value) of the hydro plants. Both the variable cost declared by the thermal units and the energy value declared by the hydro units have a regulated cap. Additional information given to CAMMESA by the generators includes: Capacity, efficiency, maintenance plan, internal consumption, availability, gas quota, fuel availability, fuel prices, reservoir characteristics, historical and predicted river inflows, down stream restrictions and water values. CAMMESA determines the generation schedule, and the marginal cost of the last unit dispatched determines the market clearing spot price, which is the basis for both the payments to generation companies and the price paid by distribution companies and large users. Dispatched generation companies receive an ex-post payment based on hourly prices at every network location. They are paid by supplied energy and capacity. The capacity price is defined by the Secretariat of Energy. Distribution companies pay a seasonal stabilized wholesale price arising from CAMMESA's "ex-ante" review based on the average marginal price foreseen in the next season. Should these prices deviate from actual dispatch, they will be compensated in the next season.

Reference: [26]

### 3.6 Differences and similarities of the market solutions

Common features:

- Distribution and transmission are identified as natural monopolies, which implies the need of regulations of these services
- Competition in supply
- All countries has free third party access to the transmission network, as they has identified this as a necessary condition for competition

Differences:

- In Norway separate accounting is required for vertically integrated companies. In

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Argentina vertical integration is not allowed
- Argentina has not introduced competition in retail sales
- Argentina and UK has implemented a pool model with centralized scheduling, whereas Norway/Sweden and California has a voluntary power exchange with simple matching of supply demand bids

For a comparative analysis of the reforms in Norway and the UK see [8] or [22].

Table 3.1. Characteristics of the UK and the Nordic electricity markets

<table>
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<td>Central Scheduling</td>
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<td>Mandatory pooling + bilateral financial trade</td>
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Chapter 4

Decentralized optimization

This chapter presents a new electricity wholesale market that matches supply and demand bids, and simultaneously secure enough spinning reserve. This market involves no central optimization, all optimization is done locally by each unit. A power exchange is used as an organized marketplace.

4.1 Introduction

In this work, it is chosen to study a model with decentralized optimization, i.e. each unit will optimize its production against the prices in the market. The prices will be set by matching supply and demand bids.

The operation of the electricity system is divided into two roles:

- System operator: Operates the transmission network, procures ancillary services and performs real-time balancing of load and generation. The system operator determines the level of spinning reserve needed to ensure reliability of the system. Spinning reserve will be purchased at the power exchange
- Power exchange: Operates the spot market. A neutral market place for sales and purchase of electricity (electric energy and spinning reserve)

Real-time operation is not considered here, the spot market of the power exchange is.

The work presented here is based on the theory of perfect competition, and the conditions for perfect competition in the short run is stated in the following section.

4.1.1 Perfect competition

The model of price determination under perfect competition was originally developed by Alfred Marshall in the late nineteenth century.
In short-run analysis the number of firms in an industry is fixed. However, firms are able to adjust the quantity they are producing in response to changing conditions.

**Perfect competition:** A *perfectly competitive industry* is one that obeys the following assumptions:

1. There are a number of firms, each producing the same homogenous product
2. Each firm attempts to maximize profits
3. Each firm is a price taker: It assumes that its actions have no effect on market price
4. Prices are assumed to be known by all market participants - information is perfect
5. Transactions are costless: Buyers and sellers incur no cost in making exchanges

Once price is determined in the market, each firm and each individual treat this price as a fixed parameter in their decisions. Although individual firms and persons are impotent in determining price, their interaction as a whole is the sole determinant of price.

A market with perfect competition will result in an allocation of resources that maximizes the sum of supplier profit and demander surplus; i.e. maximizes social surplus.

Reference: [23], chapter 15.

### 4.2 Shortcomings of the Nordic electricity market

Of the presently deregulated electricity markets, decentralized optimization is closest related to the Norwegian model. It is therefore pointed out what the main shortcomings of the system in Norway are, if the model should be used in a general hydrothermal environment:

- Decoupling in time, each hour is treated independently from the others, when the optimal solution indeed depend upon both past, present and future. Bids are delivered for each of the 24 hours of the next day, and a price is calculated for each hour not considering what happens in the other hours. This can lead to solutions that are infeasible:
  - ex. The bids from a utility which owns only one generator, a thermal unit with minimum down time of four hours, results in generation in hours 3 and 6, but not in 4 and 5. Obviously an infeasible solution.
- Reserve power is not considered in the market clearing process. This is currently not a problem in the Norwegian market, but is expected to become one in the near
future as Norway get closer connected to the rest of Europe which is dominated by thermal units. In a system with more thermal production, spinning reserve is a scarce resource.

### 4.3 Proposal of a new electricity market

The market model suggested here is based on the theory of perfect competition:

- All participants are assumed to behave rationally, i.e. seeks maximum utility/profit
- Each buyer and seller assumes that their decisions have no influence on the prices (price-takers)

One hour is chosen as the time interval.

The commodities are electric energy and reserve power within the hour (the chosen time interval). A price of energy (the $/MWh rate) and a price of reserve power ($/MW) will be set for each hour.

There has been three guidelines in this work:

1. Simplicity
2. All optimization should be done by the local entity
3. No discrimination between different types of generation. All units shall be treated equally

The reason for simplicity (first guideline) is that the participants have to understand the market rules for the market to function properly. The more complex the market rules are, the more likely it is that it will not operate as intended. The second guideline is a direct consequence of the chosen path to follow (decentralized optimization), and the last guideline should be obvious.

Financial instruments such as future contracts and options contracts are considered purely as tools for risk management (and speculation), and are not treated here. Futures and options contracts can be traded decoupled from the physical operation of electricity supply system.

#### 4.3.1 Information

One of the conditions for perfect competition is perfect knowledge; prices should be
known by all participants.

Information private to each unit (or more correctly, the company that owns it):
- Bids
- Unit fuel cost function and other plant characteristics (start-up costs, minimum up/down time etc.)
- Resulting quantities for each unit

Information submitted to the Power Exchange
- Bids
- Marginal cost function and minimum and maximum production levels

Public information:
- Clearing prices of energy and reserve power
- Market clearing procedure
- Total scheduled supply/demand in each hour

With the clearing prices known, each buyer and seller can check if their quantity of consumption/production is correct according to their bid.

### 4.3.2 Network regime

Network tariffing and handling of network restrictions are important elements of an electricity system. However, due to the limited time frame of this work, some sacrifices has to be done, and it is chosen to neglect network restrictions and network tariffing.

A one bus bar model of the electricity system is used. All generation units and all load points are assumed to be connected to the same bus bar by lossless connections (see fig. 4.1). Network tariffing and handling of network restrictions are identified as topics for future research.
4.3.3 The Power Exchange

It is chosen to have a power exchange (PX) as an organized marketplace for electricity. The responsibilities of the PX is to match supply and demand bids, and to satisfy the spinning reserve requirement.

Due to constraints like minimum up time (MUT), minimum down time (MDT), start-up costs as a function of down time, and volume constraints one cannot treat each hour independently of the others. Therefore, the proposed model is based on an hour by hour bidding and clearing process, i.e. hour \( t \) is cleared before bids for hour \( t+1 \) are collected. This way, the market participants know their commitment in hour \( t \) before they submit their bids for hour \( t+1 \); the history is known. The units use a price forecast to account for the future. All intertemporal constraints can then be treated locally, by each unit.

Two prices will be set for each hour:

- A price of electric energy ($/MWh)
- A price of reserve power ($/MW)

The market clearing process handles the spinning reserve requirement and the power balance simultaneously. The system operator sets the required level of spinning reserve as to ensure the security of the system. Reserve power can be considered a public good (see definition below), and the costs of providing it should be shared by all beneficiaries (i.e. all loads).

Fig. 4.1 One bus bar model of the electricity system
**Public Good:** A good is a (pure) *public good* if, once produced, no one can be excluded from benefiting from its availability. [23]

Power producers are paid for the energy they produce within the hour (MWh) and the amount of reserve power they provide, at the market clearing prices for that hour. The price of energy in hour t is called $\lambda_t$, the price of reserve power in hour t is called $\mu_t$. All production is paid the same prices; the market clearing prices of energy and power ($\lambda_{c,t}$ and $\mu_{c,t}$). Income for a unit i that is up and running in hour t is then:

$$
\text{Income}_i = \lambda_{c,t} \cdot p_i + \mu_{c,t} \cdot (P_i^{\text{max}} - p_i)
$$

(4.1)

A unit which is not running will not get paid.

Each unit (i.e. each generator) provides a bid, which is a monotonically increasing price-quantity curve (all supply bids must be monotonically increasing to guarantee a single equilibrium point). The bid describes the minimum energy price ($\lambda$) the unit requires for producing, with the price of spinning reserve ($\mu$) equalling zero. Figure 4.2 shows an example of a bid submitted by a single generator. For prices below $\lambda_1$ production will be zero, for prices between $\lambda_1$ and $\lambda_2$ the production will be $P_i^{\text{min}}$, for prices between $\lambda_2$ and $\lambda_3$ production is given by the line AB, and for prices above $\lambda_3$ the production will be $P_i^{\text{max}}$. For a price equalling $\lambda_1$, production will be either 0 or $P_i^{\text{min}}$. Both production levels will yield the same profit; zero. $\lambda_1$ is the indifference price, the price for which it is indifferent of running vs. not running.

![Fig. 4.2 Example of a bid submitted by a single generator](image-url)
To provide spinning reserve, each generating unit must deliver a bid. With two products (energy and spinning reserve) a three-dimensional bid is necessary to declare production levels for all possible levels of $\lambda$ and $\mu$. This three dimensional bid can be calculated with the supply curve for $\mu = 0$ and the marginal cost function known. The marginal cost function is used for minor adjustments later, and it is therefore chosen that the generators submit their marginal cost function in addition to the bid for $\mu = 0$. The marginal cost function, $MC_i(p_i)$, is the derivative of (2.5):

$$MC_i(p_i) = \frac{d}{dp_i}(FC_i) = b_i + 2c_i p_i \quad \text{for} \quad P_i^{\text{min}} \leq p_i \leq P_i^{\text{max}} \quad (4.2)$$

To summarize, each generator will submit the following information to the PX:

- The bid, a monotonically increasing price-quantity curve, which states the production level for the unit for all possible values of $\lambda$ with $\mu = 0$
- Marginal cost function (includes information about minimum and maximum production levels of the unit)

Demanders will be required to submit a price-quantity curve only.

The PX collects all supply and demand bids. The demand bids are vertical if demand is assumed to be independent of price. For a given hour the PX calculates the prices of energy and power that clears the market (power balance and enough reserve power). Prices and quantities are then set, and bidding on the next hour starts.

Procedure for the PX:

1. $t = \text{starthour}$
2. Collect bids for hour $t$.
   - Information collected for each unit:
     - Price-quantity curve (the bid)
     - Marginal cost function
3. Calculate the prices of energy and reserve power that clears the market (described in section 4.3.6)
4. Calculate quantities for each bidder (described in section 4.3.5)
5. Publish clearing prices of energy and reserve power. Inform bidders of their energy and spinning reserve requirements for hour $t$
6. $t = t + 1$, go to 2.
4.3.4 Optimization by the single firm

Bid routines for generators are developed here.

According to the theory of perfect competition, the following assumptions are made [23]:

- Each firm attempts to maximize profits.
- Each firm is a price taker; i.e. it assumes that its actions have no effect on market price.

The first assumption is a fairly good assumption we can make about the behavior of the firms. The price taker assumption is rational when the number of firms is high.

To be able to account for minimum production of units and system reserve, it is chosen that each generator unit calculates its own bid. A firm that owns several generators should submit one bid for each generator to the power exchange.

For deciding production levels for different prices, each unit must have a description of its costs. In short-run planning only the variable costs are of interest, since the firm has no influence on fixed costs.

**Short-Run Fixed and Variable Costs:** Fixed costs (FC) are costs associated with inputs that cannot be varied in the short run. Variable costs (VC) are costs of those inputs that can be varied in order to change the firm’s output level.

In the short run a price-taking firm will produce the level of output for which the marginal cost (MC) equals the price. For prices below average variable cost (AVC), however, the firm will choose to produce no output. If the income is lower than the variable costs, it will be more profitable to stop production. See [23], p.380-383, (or another textbook in microeconomic theory) for details. A supply curve for a price-taking firm is shown in fig. 4.3.

**Marginal cost:** Marginal cost (MC) or incremental cost is the additional cost of producing an additional unit.
For a typical thermal plant with AVC higher than MC at all production levels, the supply curve is given by the heavy lines in fig. 4.4. For prices lower than AVC the quantity offered will be 0, and for higher prices the quantity offered will be $P_{max}$.

Fig. 4.4 Supply curve for a typical thermal plant with variable cost described by a second order polynomial


**Bid calculation**

It is assumed here that the unit production costs can be described by a second order polynomial. This is the most common way to describe thermal units. Hydro units with enough reservoir capacity to assume a constant water value within the planning horizon (24 to 168 hours) can also be described this way.

It is easily understood that a bid only based on average variable costs and marginal costs will not give the optimal solution for the unit since it has start-up costs, minimum up time and minimum down time, etc.

The history is known, so the unit knows if it is in a “cannot run” or “must run” situation (due to minimum up/down time constraints or maintenance or some other reason). If the unit cannot run, it does not give any bid. If the unit must run, it will offer $P_\text{min}$ as long as the price is below the marginal cost at $P_\text{min} = b + 2c_\text{P}_\text{min}$. For higher prices, the quantity offered is given by the marginal cost function (4.2).

If the unit is not in a “cannot run” or “must run” situation, the minimum price that the unit demands to run must be calculated.

Since the market clearing process is decoupled in time, the units must take the time-coupling into account when making the bids. This is dealt with using a price forecast. As a first approach, the prices are assumed known for future hours.

With the prices for future hours known, the impact on future profits from running vs. not running (in the hour the bid is to be calculated for) can be calculated, $I_{\text{diff},i}$. This is done by running a backwards dynamic programming (DP) routine (see appendix A for details).

\[
I_{\text{diff},i} = \text{future profits if running} - \text{future profits if not running}
\]  

A unit which is running but can be shut down, will then correct its bid for the next hour based on the impact on future profits from running vs. not running in this hour. A unit which can be started will do the same, and will also include start up costs. The argument for this is that the difference in future income caused by a decision of production level in a specific hour should be accounted for in that hour, and not in the future (at the time of payment).

Example: A unit is running in hour t-1, and will make $500 more in all future hours
(hours t+1 and beyond) if it runs in hour t than if it does not run in hour t. It can suffer a loss of up to $500 in hour t and still be running. For greater losses it will shut down.

Thus, the total costs which must be covered in hour t is the sum of fuel costs and start-up costs, minus the difference in future income of running vs. not running in hour t ($I_{diff,t}$):

$$TC_i = a_i + b_ip_i + c_ip_i^2 + SC_i - I_{diff,i} \tag{4.4}$$

With a price of reserve power equalling zero, the income for the unit i is: $p_i \lambda$. The price which is to be calculated, is the minimum energy price the unit requires in order to produce in the given hour, and is called MinPrice. The unit is not willing to lose money, so the income must be higher than the total costs to cover (4.4):

$$\lambda p_i \geq a_i + b_ip_i + c_ip_i^2 + SC_i - I_{diff,i} \tag{4.5}$$

MinPrice is the minimum system energy price for which the unit does not loose money (break-even). Unit i will thus produce if the energy price is above (or equal) to:

$$\text{MinPrice}_i = \frac{\text{Min} (a_i + b_ip_i + c_ip_i^2 + SC_i - I_{diff,i})}{p_i} \tag{4.6}$$

For prices below MinPrice, the quantity offered will be zero. If MinPrice is lower than marginal cost at $P_i^{\text{min}}$, $P_i^{\text{min}}$ will be offered for prices between MinPrice and marginal cost at $P_i^{\text{min}}$. For quantities where marginal cost exceeds MinPrice, the quantity offered is given by the marginal cost function.

Fig. 4.5 shows the resulting bid when the calculated MinPrice is lower than the marginal cost at $P_i^{\text{min}}$. Fig. 4.6 shows the resulting bid when the calculated MinPrice is higher than the marginal cost at $P_i^{\text{min}}$. Fig. 4.7 shows the bid when MinPrice is between marginal cost at $P_i^{\text{min}}$ and marginal cost at $P_i^{\text{max}}$. Note that the bids are not continuous. The dashed lines indicate quantities that can be produced if paid extra, which will happen when a price of reserve power is set. See the next section too see how the bid is interpreted with a price for energy and reserve power.
Fig. 4.5 Bid when MinPrice is lower than marginal cost at $P_i^{\text{min}}$

Fig. 4.6 Bid when MinPrice is higher than marginal cost. The marginal cost function is also shown (MC).
4.3.5 Quantity determination with price on energy and reserve power

Maximizing income (given by (4.1)) minus fuel costs will maximize profits for a unit that is set to run:

\[
\text{Max} \quad I_i = \lambda_c \cdot p_i + \mu_c \cdot (P_i^{\text{max}} - p_i) - FC_i(p_i)
\]

(4.7)

The solution of the maximization of this function is (MC = marginal cost):

- If \(MC_i(p_i^{\text{min}}) > (\lambda_c - \mu_c)\), \(p_i = p_i^{\text{min}}\)
- If \((\lambda_c - \mu_c) > MC_i(p_i^{\text{max}})\), \(p_i = p_i^{\text{max}}\)
- Otherwise production is given by \(MC_i(p_i) = \lambda_c - \mu_c\)

With a price on both energy and reserve power, the marginal cost function gives the information of the production quantity of the unit if it is set to run. With the bid it can be decided whether it will produce this quantity or not.

Procedure to calculate quantity

A two-step procedure is used to calculate the production of a unit, given the market clearing prices \((\lambda_c \text{ and } \mu_c)\) and its bid and marginal cost function:

\[
\text{Fig. 4.7 Bid when MinPrice is between marginal cost at } p_i^{\text{min}} \text{ and marginal cost at } p_i^{\text{max}}. \text{ The marginal cost function is also shown (MC).}
\]
1. Given $\lambda_c$ and $\mu_c$, find the production ($p_{r,i}$) given by the marginal costs (4.2) with the unit set to run:
   - If $MC_i(p_{i,\text{min}}) > (\lambda_c - \mu_c)$, $p_{r,i} = p_{\text{min}}$
   - If $(\lambda_c - \mu_c) > MC_i(p_{i,\text{max}})$, $p_{r,i} = p_{\text{max}}$
   - Otherwise production is given by $MC_i(p_{r,i}) = \lambda_c - \mu_c$

2. Check if prices $\lambda_c$ and $\mu_c$ are high enough for the unit to run:
   - Income = $\lambda_c p_{r,i} + (p_{i,\text{max}} - p_{r,i}) \mu_c$
   - For the calculated $p_r$, find the corresponding bid-price. If this is on a flat segment of the bid (dotted lines in fig. 4.6 and fig. 4.7), the right hand side quantity of this segment is used ($p_{b,i}$) with its corresponding price ($\lambda_{b,i}$). Income requirement ($I_{\text{req},i}$) for the unit to run at $p_{r,i}$ is $p_{b,i} \lambda_{b,i}$ minus saved production cost of running at $p_{r,i}$ instead of $p_{b,i}$ (= integral of marginal cost (4.2)):
     \[
     I_{\text{req},i} = p_{b,i} \lambda_{b,i} - \frac{MC_i(p_{r,i}) + MC_i(p_{b,i})}{2} \cdot (p_{b,i} - p_{r,i})
     \] (4.8)
   - If income > $I_{\text{req},i}$, the production quantity of the unit is set to $p_r$. Preferred production $p^* = p_r$
   - If income < $I_{\text{req},i}$, the production quantity of the unit is set to zero. Preferred production $p^* = 0$
   - If income = $I_{\text{req},i}$, the unit is indifferent of running vs. not running (with the given prices), and it is defined as an indifferent unit. Preferred production $p^* = \frac{S}{c}$

An example of this procedure is given in appendix D.1.

### 4.3.6 Market clearing

Some symbols are needed in the explanations here:
- $P_1$ = Sum preferred production ($p^*$) for all units except indifferent ones
- $P_2$ = Sum preferred production ($p^*$) for all units
- $P_3 = P_2 + \text{sum reserve of units with } p^* > 0$
- $P_D$ = Sum demand
- $R$ = Reserve requirement

Market clearing involves the process of finding the clearing prices of energy and reserve power. There can be an infinite number of ($\lambda$, $\mu$) pairs that clears the market (i.e. supply equals demand, and spinning reserve requirement satisfied). The market clearing prices are defined as the minimum $\mu$ that satisfies the spinning reserve requirement and the cor-
responding price of energy $\lambda$. The clearing prices are called $\lambda_c$ and $\mu_c$.

Since the bids are not continuous, exact matching of supply and demand is not always possible. The clearing price of energy is defined to be found if $P_1 \leq P_D \leq P_2$.

An iterative procedure is used to calculate these clearing prices. This procedure is shown in fig. 4.8 and fig. 4.9.

With the clearing prices found, a preliminary schedule is set:
- Production of all units but the indifferent ones is set to $p^*$
- As many units as needed of the indifferent units to satisfy the spinning reserve requirement and make sum production equal to or greater than demand are committed (in no particular order). Production of the committed indifferent units is set to $p^*$.

With the clearing prices and the according preliminary schedule found, an adjustment has to be made to make the schedule feasible. Sum production of the preliminary schedule will be equal or higher than demand (due to discontinuous bids). If sum production is higher than demand, production of the most expensive (highest marginal cost) units are reduced to make supply equal to demand. The resulting schedule is final.

The units will be paid according to this final schedule. However, if the final schedule is different from the preliminary one, extra payments are necessary. See section 4.3.7 for how these extra payments are calculated.
\( \mu = 0 \)  
Delta = start-increment  
foundMax = FALSE  
precision = small value

For this value of \( \mu \), find \( \lambda \) that matches supply and demand, and the value of \( P_3 \)

\[ \mu = 0 \text{ and } P_3 \geq P_D + R \]

\[ P_3 \geq P_D + R \]

If foundMax, set \( \Delta = \Delta / 2 \)  
If \( \Delta < \text{precision} \), set \( \Delta = \text{precision} \)  
\( \mu = \mu + \Delta \)

Clearing prices found  
\( (\lambda_c \text{ and } \mu_c) \)

\[ \Delta = \Delta / 2 \]  
\[ \mu = \mu - \Delta \]  
\[ \text{foundMax} = \text{TRUE} \]

Fig. 4.8 Flowchart for calculation of clearing prices
Fig. 4.9 Flowchart for calculation of clearing price of energy for a fixed price of reserve power

1. \( \mu \) fixed
2. \( \Delta = \text{start-increment} \)
3. \( \lambda = \Delta \)
4. foundMax = FALSE

For all units, find \( p^* \) and identify indifferent units. Calculate \( P_1 \) and \( P_2 \)

- If \( P_1 \leq P_D \leq P_2 \), go to No
- If \( P_2 \geq P_D \), go to No

If foundMax, set
- \( \Delta = \Delta / 2 \)
- \( \lambda = \lambda + \Delta \)

No

Clearing \( \lambda \) found for fixed \( \mu \)

Yes

\( \Delta = \Delta / 2 \)
\( \lambda = \lambda - \Delta \)
foundMax = TRUE
4.3.7 Payments for deviation from preferred production, \( p^* \)

Those units that are set to produce a quantity other than \( p^* \) must be paid to do this, so that their net profits are unchanged from the preliminary schedule (the units should not be made worse off by the adjustments of the schedule).

Income for preliminary schedule:

\[
I_1 = \lambda_c \cdot p^* + \mu_c \cdot (P^{Max} - p^*) \tag{4.9}
\]

Income for final schedule:

\[
I_2 = \lambda_c \cdot p_f + \mu_c \cdot (P^{Max} - p_f) \tag{4.10}
\]

Saved costs (given by integrating marginal costs):

\[
C_1 = \frac{MC(p_f) + MC(p^*)}{2} (p^* - p_f) \tag{4.11}
\]

Extra payment is equal to the reduction in income minus the reduction in costs:

\[
Extra = I_1 - I_2 - C_1 \tag{4.12}
\]

This extra payment will be paid for all units with final schedule different from initial schedule.

It is assumed that the costs associated with these extra payments will be very small compared to total costs.

4.4 Bilateral trade

One question that arises is whether or not all power should be sold through the PX. With the proposed market, there is nothing that makes it necessary to sell all power through the PX. In the present Norwegian market it is also allowed to make bilateral contracts.

In this system with decentralized optimization, trading on the power exchange is not mandatory. Since the production companies can be paid for making production available (with a non-zero price of reserve power, \( \mu \)), producers will profit on making bids to the PX.

Example 1:
A 100 MW unit has a bilateral contract of 60 MW in all hours. This does not prevent it from participating in the power pool by declaring the remaining 40 MW of production capacity available at marginal cost. By bidding on the PX, the unit has potential for additional income.

Example 2:
A unit which have sold all its production by bilateral contracts need not bid to the PX at all.

### 4.5 Bid calculation for hydro units

Whereas fuel cost is a fairly well known parameter in thermal systems, there is no pre-assigned value associated with a marginal amount of stored water. Due to this fact, the short-term hydro scheduling is a problem of how to use the available water volume for generation. [11]

Long-term reservoirs are meant to take advantage of variations in prices due to seasonal and yearly variations of load and inflow. The optimization of hydro units is therefore usually divided into a long-term problem and a short-term problem, since optimization with a horizon of up to several years with a time step of one hour would be an enormous task taking the stochastic nature of inflow into account. In the long term problem a week is a normal time step.

The coupling between the long-term and short-term problem is normally either through water values (expected value or incremental worth of water) or by water discharge quantities for the week.

For long-term reservoirs, the water values can be treated as fixed in the short-term scheduling. When assigning water values to hydro units, the problem is equivalent to the thermal problem, and bid calculation can be done according to section 4.3.4.

In river systems, discharge from one power station can be inflow to the next. This hydraulic coupling means that the production scheduling of one unit cannot be done independently from the other units in the same river system.

A full treatment of hydro units is not possible within the limited time of this project.
4.6 Practical implementation

The bidding and clearing process of the proposed market is summarized in fig. 4.10. It is an infinite loop, as it should work for an indefinite number of future hours. Each iteration in this flowchart must be completed in one hour (if the planning of one hour take longer than one hour, time will run out). If each iteration takes one hour, this process would require 24 hour a day operation. To finish the planning of 24 hours in a working day (8 hours (even saturdays and sundays)), the time in one iteration should be no more than 20 minutes. With automated routines for bidding, and good routines for information exchange (between the bidders and the PX), this should be possible. The calculations needed in the bid calculation (carried out by each unit) and the market clearing (carried out by the PXO) will not take more than seconds on a modern computer, so information exchange will probably be the most time consuming task. In addition, each unit has to

---

**Fig. 4.10** Flow chart of market model.

(PX = Power eXchange)
generate a price forecast (used in the bid calculation) for a number of hours ahead. How many hours the price forecast must be for depends on the unit characteristics. E.g., a unit with high start-up costs may have to consider a longer period than a unit with no or small start-up costs.

The lead time should be as low as possible, to reduce uncertainties in load, plant availability etc.

A market that clears one hour per hour has the advantage that planning can be done close to operation, thus reducing uncertainties. The disadvantage is the need for 24 hour manning (if the bidding process is not automated). With all planning done within the working day, the minimum lead time for the last hour planned will be 16 hours.

4.6.1 Alternative solutions

The simplest version of the market, and thereby appealing, is the one already described; A one iteration only procedure, with moving horizon.

An alternative is to do this process for one day (24 hours), use the resulting prices to update the price forecast and start over again. The number of iterations is limited by the available time (24 hours to schedule the next 24 hours), but a few iterations (two or three) should be possible.

Algorithm for running more than one iteration:
1. Iteration = 0
2. t = 1, iteration = iteration + 1
3. All units calculate bids for hour t and submit them to the PX
4. PX calculates clearing prices and quantities, and publishes the prices
5. t = t + 1
6. If t > 24 and iteration > max. no. of iterations then go to 8.
7. If t > 24 then go to 2 else go to 3
8. Publish clearing prices for the whole day, and inform each bidder of their energy and reserve power requirements

Another possibility is to for the PX to publish a price forecast (which the units can choose to use or not), and if the resulting clearing prices deviate more than some predefined level, a new iteration is run. This way, the units can not speculate by giving
incorrect bids in the first iteration, knowing that the bids for the first iterations in no way are binding.
Chapter 5

Simulated Competition

To evaluate the quality of the proposed market, a computer simulation program has been developed. This program simulates the proposed market according to the bidding and clearing rules described in the previous chapter.

5.1 Simulated competition (SimCom)

A computer program based on the proposed market is developed. The program is called Simulated Competition (or SimCom for short). SimCom is written in the Fortran 90 programming language.

The critical factor in the proposed market is the price forecast. The price forecast consists of the expected price of energy and the expected price of reserve power for all hours of the optimizing horizon. A computer simulation program allows us to run several iterations, where the price forecast is updated in each iteration. An initial price forecast must be set, but it is not critical that it is a good one, as it will converge to a better price forecast. The old price forecast together with the clearing prices (resulting from that old price forecast) is used to calculate the new price forecast (one price of energy and one price of reserve power for each hour of the optimization period):

$$\text{new price forecast} = \text{old price forecast} \cdot (1-\alpha) + \text{resulting clearing price} \cdot \alpha (5.1)$$

$\alpha$ is an update factor, and is usually set to 0.5, giving the new price forecast as the mean of the old price forecast and the clearing prices resulting from that price forecast. All units use the same price forecast.

SimCom can be used to solve the scheduling problem with minimum up/down times, time varying start-up costs, minimum and maximum production levels and spinning reserve requirement taken into account.
5.1 Simulated competition (SimCom)

Figure 5.1 shows the flowchart for SimCom:

- Set initial price forecast
- Set update factor $\alpha$
- Set $t=0$

$t = t + 1$

- $T > \text{horizon}$
  - No: For this hour:
    - Calculate bids
    - Find clearing prices ($\lambda_{c,t}$ and $\mu_{c,t}$)
    - Find production schedule
  - Yes: Calculate production costs
    - Calculate dual value

- Solution good enough
  - No: Update price forecast
    - Set $t = 0$
  - Yes: Write results

Fig. 5.1 Flowchart for SimCom
**horizon** is the number of hours to simulate (usually between 24 and 168 hours)

$\lambda_{c,t}$ is the clearing price of energy in hour $t$ (in $/\text{MWh}$)

$\mu_{c,t}$ is the clearing price of reserve power in hour $t$ (in $/\text{MW}$)

Bid calculation is described in section 4.3.4, market clearing (price and quantity determination) is described in section 4.3.6. The dual value is described in section 5.1.1.

In the bid calculation, the hours considered when calculating $I_{\text{diff},i}$ is from $t+1$ to the last hour of the optimizing horizon. Recall that the difference in future income for running vs. not running hour $t$ has to be found ($I_{\text{diff},i}$ in (4.6)) to calculate the bid for hour $t$. See the following example:

**Example:**
Optimizing horizon is 24 hours. For hour 1, the price forecast for hours 2-24 is used when calculating the bids for all units for that hour. For hour 2, the price forecast for hours 3-24 is used. In hour 23, the future is hour 24. In hour 24 there is no future to consider, since hours beyond the optimizing horizon is not considered, and thus $I_{\text{diff},i} = 0$.

There is no explicit convergence criterion in SimCom. When running the program, the simulation is usually stopped after 20 iterations.

A paper about this method, called 'Thermal Power Generation Scheduling by Simulated Competition', is accepted for publication in an IEEE Transactions. [14].

### 5.1.1 Dual value

Due to similarities with the Lagrangian Relaxation method (LR), the clearing prices in SimCom can be used as the multipliers in LR to calculate the Lagrangian dual value. The dual solution is not a feasible solution to the problem, but the dual value represents a lower limit of total costs. The difference between the calculated cost and the dual value represents an upper limit for the distance to the optimal solution.

With the problem definition from section 2.5, the dual function becomes (see appendix B for details):

$$
D = \sum_{i=1}^{N} \min \{ K_i(u_i, p_i) \} + \sum_{t=1}^{T} (\lambda_t \cdot P_{D,t} + \mu_t \cdot R_t) \quad (5.2)
$$
where

\[ K_i(u, p) = \sum_{t=1}^{T} FC(p_{i,t}) + \sum_{t=1}^{T} SC_{i,t} - \sum_{t=1}^{T} \left( \lambda_{i,t} \cdot p_{i,t} + \mu_{i,t} \cdot u_{i,t} \cdot (P_{i}^{\text{max}} - p_{i,t}) \right) \]  \hspace{1cm} (5.3)

\[ u_{i,t} = 1 \text{ if unit } i \text{ is running in hour } t, \text{ else } 0 \]

The value of the dual function is calculated in each iteration of SimCom using the clearing prices as multipliers.

### 5.2 Summary

SimCom can be used to solve a standard scheduling problem, as formulated in section 2.5.

**Input to SimCom:**

- Unit data for all units (minimum production, maximum production, minimum up time, minimum down time, fuel costs described by a second order polynomial, linear or exponential start-up costs, and initial condition (on/off and for how long))
- Optimization horizon (number of hours to simulate)
- Load and reserve requirement for each hour of the optimization period (only firm load is implemented in SimCom so far)
- Initial price forecast
- Update factor \( \alpha \) (determines how the price forecast for the next iteration is set, see equation (5.1))

With the input data given, SimCom runs for a given number of iterations (usually 20). For each new iteration, the price forecast is updated.

**Output from SimCom:**

- Production cost (sum of fuel costs and start-up costs)
- Production plan (production level for each unit in each hour)
- Dual value (representing a lower limit of production cost, NOT a feasible solution)
- Resulting prices of energy and reserve power for each hour
SimCon

Reading unit data from: datfile\tern40.dat
Reading load data from: datfile\load40.dat
Optimizing horizon: 48 hours

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<th>Iteration</th>
<th>Fuel cost</th>
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<th>Total cost</th>
<th>Dual value</th>
<th>Cap</th>
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</table>

Fig. 5.2 Sample screen output while running SimCom
5.2 Summary
Chapter 6
Test simulations

SimCom, the computer program that simulates the proposed market, has been tested on several cases, which results are shown here. The results provide information about the quality of SimCom as a program for solving the single-owner scheduling problem, and about the quality of the proposed market. The results will be discussed in the next chapter.

6.1 Introduction

To be able to evaluate the quality of the proposed market and of SimCom, simulations with SimCom has been done. Simulations will provide information about convergence properties, and the sensitivity of the assumption of known future prices can be examined.

The simulations are done to study the quality of the two different contributions of this thesis:

1. SimCom as an optimization tool for the single owner scheduling problem/central scheduling problem
2. The quality of the proposed scheduling market

Test results from 3 different cases is reported here: A 10 unit case and a 110 unit case, both with data from [24], and a 9 unit case with known optimal solution. Optimization horizon is 24 hours for the first two cases, and 168 hours for the third. Complete data for all cases can be found in appendix C.

For comparison, the Unit Decommitment procedure described in [19] is implemented with Criterion 1 as the decommitment criterion (see [19]). This Unit Decommitment procedure starts with all units committed, and then iteratively shuts down (in all or some hours) one by one unit until no more cost savings are possible. A heuristic rule (Criterion
1) is used to decide which unit to shut down in each iteration.

A lagrangian relaxation based program has also been available; SHARP. SHARP is described in [17].

The schedule costs reported for solution methods other than SimCom, SHARP and Unit Decommitment are from [24]. These are marked by a star (*) in the tables.

The initial forecast for price of reserve power is set to zero in all simulations.

The SimCom method is programmed in Fortran 90 and run on a PC with a Pentium 133 MHz processor.

The SimCom method and the economic results of test systems 1 and 2 is presented in a paper that has been accepted for publication in IEEE Transaction on Power Systems [14].

### 6.2 SimCom

#### 6.2.1 Test system 1 (10 units, 24 hours)

Table 6.1 shows a comparison of SimCom best solution with the best solutions found by other optimizing techniques. The Lagrangian dual value (see previous chapter) is also reported. Table 6.2 shows the best unit commitment plan generated by SimCom and the best generated by SHARP.

<table>
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<tr>
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<th>Schedule cost ($)</th>
<th>% from best</th>
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</thead>
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<td>Dual value</td>
<td>47 134</td>
<td>-0.29 %</td>
</tr>
<tr>
<td>SHARP</td>
<td>47 273</td>
<td></td>
</tr>
<tr>
<td>Simulated Competition</td>
<td><strong>47 281</strong></td>
<td><strong>0.02 %</strong></td>
</tr>
<tr>
<td>Lagrangian Relaxation*</td>
<td>47 511</td>
<td>0.50 %</td>
</tr>
<tr>
<td>Hybrid genetic algorithm*</td>
<td>47 576</td>
<td>0.64 %</td>
</tr>
<tr>
<td>Normal genetic algorithm*</td>
<td>47 596</td>
<td>0.68 %</td>
</tr>
<tr>
<td>Unit decommitment</td>
<td>47 704</td>
<td>0.91 %</td>
</tr>
<tr>
<td>Artificial neural network*</td>
<td>48 293</td>
<td>2.16 %</td>
</tr>
</tbody>
</table>
Solution time for 20 iterations with this case is 6 seconds.

Table 6.2. Unit commitment plan for the best schedule generated by SimCom for test system 1 (Underlined where the solution differs from SHARP’s solution)

<table>
<thead>
<tr>
<th>Unit</th>
<th>Hours (1-24)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11111111111111111111111</td>
</tr>
<tr>
<td>2</td>
<td>11111111111111111111111</td>
</tr>
<tr>
<td>3</td>
<td>11110000000000000000000</td>
</tr>
<tr>
<td>4</td>
<td>11111111111111111111111</td>
</tr>
<tr>
<td>5</td>
<td>11111111111111111111111</td>
</tr>
<tr>
<td>6</td>
<td>00000000000000000000000</td>
</tr>
<tr>
<td>7</td>
<td>11111111111111111111111</td>
</tr>
<tr>
<td>8</td>
<td>00000000000000000000000</td>
</tr>
<tr>
<td>9</td>
<td>11111111111111111111111</td>
</tr>
<tr>
<td>10</td>
<td>11111111111111111111111</td>
</tr>
</tbody>
</table>

6.2.2 Test system 2 (110 units, 24 hours)

For this system the reserve requirement is set to 700 MW in all hours ($P_{\text{max}}$ of largest unit).

Units 101-110 are specified with shut-down costs in [24] (ranging from $15 to $75). In the calculations with SimCom, SHARP and Unit Decommitment the shut-down costs are added to the start-up costs. The maximum error caused by this is $163 (Shut-down of those units which are initially running).

Table 6.3. Comparison of SimCom best solution with other solution methods for test system 2 (110 units)

<table>
<thead>
<tr>
<th>Solution method</th>
<th>Schedule cost ($)</th>
<th>% from best</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dual value</td>
<td>3 809 231</td>
<td>-0.04 %</td>
</tr>
<tr>
<td>Simulated Competition</td>
<td>3 810 859</td>
<td></td>
</tr>
<tr>
<td>SHARP</td>
<td>3 813 057</td>
<td>0.06 %</td>
</tr>
<tr>
<td>Unit decomposition</td>
<td>3 819 282</td>
<td>0.22 %</td>
</tr>
<tr>
<td>Hybrid genetic algorithm*</td>
<td>3 826 775</td>
<td>0.42 %</td>
</tr>
<tr>
<td>Normal genetic algorithm*</td>
<td>3 854 821</td>
<td>1.15 %</td>
</tr>
</tbody>
</table>
Table 6.3 shows a comparison of SimCom best solution with the best solutions found by other optimizing techniques. Figure 6.1 shows the results in each iteration of simulations with different initial energy price forecasts (these results are also shown in tabular form in table E.1. The initial price forecast is equal for all hours.

The solution time of Test system 2 is 1 minute for a run with 20 iterations.

6.2.3 Test system 3 (9 units, 168 hours)

This is a test case with 9 units, 7 thermal units and 2 hydro units. The hydro units are assigned a water value (incremental worth of water) from the long term planning, and can therefore be treated the same way as the thermal units.

The unit data and load data can be found in section C.3. Reserve requirement is 10% of the load in all hours.

In this case the units have no minimum up or down time, and the start-up costs are constant (not dependent on downtime). The problem can then be formulated as a shortest path problem, and for a problem of this small size the optimal solution can be found. The optimal solution is found by Dijkstra’s method\(^1\), and is provided by Professor Arne

\[ \text{1. Refer to a textbook in Operations Research for a description of Dijkstra’s method, for example [10], pages 191-194} \]
Johannesen of SINTEF Energy Research. Shortest Path applied on the generation scheduling problem is described in [16].

Table 6.4. SimCom best solution vs. optimal solution for test system 3
(9 units, 168 hours)

<table>
<thead>
<tr>
<th>Solution method</th>
<th>Schedule cost (NOK)</th>
<th>% from optimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dual value</td>
<td>8 871 637</td>
<td>-1.92 %</td>
</tr>
<tr>
<td>Optimal solution</td>
<td>9 045 090</td>
<td></td>
</tr>
<tr>
<td><strong>Simulated Competition</strong></td>
<td><strong>9 051 897</strong></td>
<td><strong>0.08 %</strong></td>
</tr>
<tr>
<td>SHARP</td>
<td>9 074 970</td>
<td>0.33 %</td>
</tr>
<tr>
<td>Unit decommitment</td>
<td>9 194 557</td>
<td>1.65 %</td>
</tr>
</tbody>
</table>

The best solution found by SimCom has the following commitment plan:

- Units 1, 2 and 3 are committed in all hours
- Unit 4 is committed in hours 6-119 (all weekdays)
- Unit 8 is committed in hours 8-23, 32-47, 56-71, 80-95, 104-119, 126-142 and 150-166 (once per day)
- Unit 9 is committed in hours 11-13, 35-37, 59-61, 83-85, 107-109, 128-133 and 152-157 (once per day)

This is illustrated in fig. 6.2 (since only 3 units are switched on and off, it is possible to identify each start and stop in the figure). The only difference from the optimal solution is that in the optimal solution, the smallest of the two hydro units (unit 9) is used instead of the largest hydro unit (unit 8) in hours 17-18, 23, 41-42, 47, 65-66, 71, 89-90, 95, 113-114, 119, 126, 142, 150, 166.
6.3 Quality of the proposed market

The simplest version of the market is the one-iteration-only procedure. Because of its simplicity, it is attractive. A question is then; how good must the price forecast be to reach near-optimal solutions in the first iteration? And, are there significant improvements by running more than one iteration which justifies the increased complexity of the market?

Another question is whether the units have an operating profit or not. If a unit does not make any profits, it has no reason to participate in the market. The only reason for participating in the market is that the unit believes in opportunities to make profits.

Test case 2 is used in the simulations reported here.
6.3.1 Test with different initial price forecasts

An electricity market should be able to reach a good solution in one or just a few iterations, and results from simulation with different initial price forecast are thus shown here. See the next chapter for the discussion of the results.

Figure 6.3 shows the schedule cost in each of the three first iterations of SimCom for different initial price forecasts. The price forecast that gives the best solution in one iteration is used as a basis, and is multiplied by a scaling factor. Prices for reserve power are also forecasted, but setting the price forecast of reserve to zero makes little difference. The difference between the best and the worst solution in fig. 6.3 is 0.38%.

Calculation of the initial price forecast:

\[
\lambda(t) = c_s \cdot \lambda_{\text{best}}(t) \\
\mu(t) = c_s \cdot \mu_{\text{best}}(t)
\]  

(6.1)

Where \(c_s\) is the scaling factor, \(\lambda_{\text{best}}(t)\) is the price of energy in hour \(t\) in the price forecast that gives the best solution and \(\mu_{\text{best}}(t)\) is the price of power in hour \(t\) in the price forecast that gives the best solution (this "best" forecast is shown in fig. 6.4). The values of \(\lambda(t)\) and \(\mu(t)\) are then used as the initial price forecast.

Fig. 6.3 Schedule cost in the first iterations of SimCom for different initial price forecasts for test case 2. The price forecast that gives the best solution (in one iteration) is scaled by the "Scaling factor"
6.3.2 Do the units earn any profits?

For the units to participate in the market, they must have opportunity to make profits. The question is then: Do the units make any profits?

If the price forecast is correct (equals the resulting prices), all units will make profits since their bids are optimized for the price forecast. However, since the price forecast is uncertain, the units risk both loosing money and earning more than expected.

One iteration of SimCom is run, with the price forecast that gives the best schedule. This price forecast and the resulting prices is shown is fig. 6.4 (all units use the same forecast).

The price forecast and resulting prices is shown in tabular form in table E.3.

![Fig. 6.4 Price forecast and resulting prices (best iteration of test case 2)](image)

The payment for delivered energy is $5,208,525 and the payment for reserve power are $12,605 (0.24% of total payments). The extra payment for the difference between the final schedule and the preliminary one totals to $233 (0.0045% of total payments), which confirms that the assumption that these costs are small is valid (at least for this case).

The payments to the generators total $5,221,363 whereas the generation costs are $3,810,851. This gives an operating profit of $1,410,512. However, profits are not
equally distributed among the units, and the profit for each unit has to be checked.

Four units lose money in this case, the units are no. 14, 15, 103 and 109. Table 6.5 shows the net income in each hour for these units. Start-up (and shut-down) costs are accounted for in the hour of start-up.

Table 6.5. Income in each hour for selected units

<table>
<thead>
<tr>
<th>Hour(s)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4-17</th>
<th>18</th>
<th>19</th>
<th>20</th>
<th>21</th>
<th>22</th>
<th>23</th>
<th>24</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit 14</td>
<td>-407</td>
<td>1</td>
<td>490</td>
<td>-255</td>
<td>-170</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unit 15</td>
<td>-143</td>
<td>481</td>
<td>-257</td>
<td>-285</td>
<td>-205</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unit 103</td>
<td>155</td>
<td>-240</td>
<td>-350</td>
<td>-176</td>
<td>79</td>
<td>262</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-496</td>
</tr>
<tr>
<td>Unit 109</td>
<td>-787</td>
<td>260</td>
<td>610</td>
<td>-447</td>
<td>-523</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-888</td>
</tr>
</tbody>
</table>

These units lose money since the resulting prices are lower than they forecasted. Their behavior is optimal according to the price forecast and the resulting price in the hour of start-up, but since prices are lower than expected, they lose money.

Units 14, 15 and 103 have minimum up time of four hours, and unit 109 has minimum up time of 5 hours. This explains why they are up in hours 22 and 23. The reason for unit 103 not shutting down in hours 2 and 3 is that it initially has been up for one hour only, and therefore has to be up for three more hours (hours 1, 2 and 3).
6.3 Quality of the proposed market
Chapter 7

Discussion and further work

7.1 Introduction

There are two main contributions of this thesis:

1. A new method for decentralized scheduling and market clearing based on competitive bidding. The market clearing handles power balance and spinning reserve requirement simultaneously
2. A new algorithm for central scheduling based on simulation of the above mentioned process

This chapter discusses the quality of these two, and also identifies topics for future research.

7.1.1 SimCom as an optimization tool for the central scheduling problem

SimCom can be used to solve the single owner scheduling problem (as stated in section 2.5), and the simulations show excellent results. For test case 1, the SimCom solution is only 0.02% inferior to the solution provided by SHARP. For test cases 2 & 3, the simulated competition method (SimCom) gives better schedules than the other methods (0.06% better than the second best for test case 2, and 0.25% for test case 3). For test case 3, the optimal solution is available, and the best solution provided by SimCom is only 0.08% from the optimal solution.

Other cases differing in number of units, length of study time and system demand profile has also been tested. All these cases have shown that SimCom provides excellent performance: fast computation, fast convergence and low schedule costs.

Simulations have shown that the initial price forecast has little influence on the best solution found. Near optimal solutions are found regardless of what the initial price forecast is.
SimCom has great resemblances with the Lagrangian Relaxation (LR) method but also some differences.

Similarities with LR:

- The prices (multipliers in LR) are the only coordination signals
- The convergence properties of SimCom seem to be similar to LR; the best solution is found within a few iterations
- Since the size of units is not considered while committing units, results for small systems (only a few units) can be poor. Performance is better for large systems
- Prices (multipliers in LR) seldom stabilize completely, but end up flip-flopping between two or more sets of prices

The differences are:

- In LR the resulting commitment from the DP is binding, in SimCom only the future earnings calculated by the DP are used in calculating the bid. The commitment plan in SimCom is found by matching supply and demand bids (the demand bids being price insensitive in the simulations here due to firm load)
- One drawback with LR is that equal units are always committed equally. This is not the case with SimCom
- SimCom has one drawback: It is not checked that future loads can be covered before shutting down a unit. This could lead to infeasibilities, but has not been a problem in any of the cases tested so far
- LR solves the dual problem, and some heuristic rules are used to find a primal feasible solution. SimCom give feasible solutions directly

7.1.2 The quality of the proposed market

The simulations show that the proposed market can lead to near-optimal performance, when the units behave according to the perfect competition conditions (each firm has no influence on the market clearing price).

The critical factor is the price forecast. If the price forecast is correct, each unit will have its optimal schedule.

Optimal behavior in any given time interval is dependent on both past and future due to start-up costs and minimum up/down time. The past is always known, and the future is dealt with through the price forecast.
Simulations have shown that good schedules can be reached in just one iteration if the price forecast is not too far off. Figure 6.3 shows that for test case 2, the production cost in the first iteration are less than 0.4% higher than for the best schedule even with the price forecast 25% off. Since the gain in running more than one iteration is small, the one iteration only procedure is the most appealing due to its simpleness.

However, the units are more sensitive to the difference between the price forecast and the resulting prices. If a unit believes in high future prices, and the clearing prices are lower than expected the unit risk losing money. If the price forecast is lower than the clearing prices, the unit can lose the opportunity to make money.

In the simulations in chapter 6, all units use the same price forecast. In a real market it can be discussed whether the PX should publish price forecasts or not. The arguments against this is that the PX should be independent and neutral, and the PX will influence the bids by setting the price forecast. The arguments for a public price forecast is that it is efficient (instead of many units doing the same work one central agency makes the price forecast), and the units can independently correct the public forecast according to own beliefs. A conclusion is not drawn here, and this question is open for further discussion.

In the proposed market, discontinuous bids are allowed. The discontinuities comes from minimum production levels of units, and best operating points at higher levels of production (see figs. 4.5, 4.6 and 4.7). Due to the discontinuities in the bids, exact matching of supply and demand is not always achieved in the preliminary schedule. Adjustments are then made to this preliminary schedule to reach the final schedule where supply equals demand. With a large number of units, these adjustments will be small compared to total production, and thus the economic effect will be small.

Because spinning reserve is included in the market clearing process, a bid must be included for each unit. The market clearing then gives the resulting quantities for each unit. A production company that owns several units need not use these quantities as a final production schedule, since only the sum production and sum reserve for the units it owns are binding for the company. The company can reschedule the production any way it pleases.
7.2 Suggestion for future work

One of the most interesting topics for future work would be to develop bidding routines for hydraulically coupled hydro units (several hydro units in the same river system). With bidding routines for hydro units, it would be possible to solve the whole hydro-thermal scheduling problem.

Network restrictions (transfer limits and reliability) is not treated in this work. This is another topic for future research.

Other topics for future work are:

• Investigate market imperfections. Market power. Gaming.
• How would a stochastic price forecast influence the bids?
• Tests on actual cases
Chapter 8

Conclusions

A new electricity spot market is proposed. This market balances power production with demand and simultaneously ensures enough spinning reserve. Routines for bidding and market clearing are developed. The unit characteristics and restrictions that are handled are: Minimum and maximum production levels, fuel cost function, start-up costs, minimum up time and minimum down time.

A computer program that simulates the proposed market has been developed. The simulations show that it is possible to reach economically efficient (i.e. near-optimal) generation plans through the proposed market. The critical factor is the quality of the price forecast which is used in the bid calculations. With a reasonably good price forecast good solutions are reached already in the first iteration of SimCom. If implemented in a real market, a one iteration procedure would be preferable due to its simpleness.

A new method for solving the single owner short term scheduling problem is presented. Simulation results show that it produces near optimal production schedules, and computation time is low (1 minute for scheduling 110 units for 24 hours on a Pentium 133 PC).
References


Appendix A

Dynamic Programming (DP)

This is not a description of dynamic programming, but an explanation of how it is used in the bid calculation (chapter 4.3.4).

A.1 Backward Dynamic Programming applied to single unit optimization

Dynamic programming is a technique for finding the optimal route (and the associated optimal value) in a decision tree. If you are not familiar with DP, see a textbook in operations research, for example [25], chapter 10.

Dynamic programming is here used to find the difference in future income for the unit resulting from the decision of running vs. not running in the hour the bid is to be calculated for (i.e. the optimal profit for two different initial states must be calculated). To use DP, the problem has to be divided into stages, and a return function and transition function must be set.

With a given set of prices, each unit can find its optimal production plan. The unit wants to maximize total profits for the planning horizon (income (4.1) minus fuel costs and start-up costs):

\[
\text{Max } I_i = \sum_{t=1}^{T} \lambda_t \cdot p_{i,t} + \mu_t \cdot (p_{i\max} - p_{i,t}) - FC_i(p_{i,t}) - SC_{i,t} \quad (A.1)
\]

With respect to its restrictions:

- Unit rated minimum and maximum production
- Minimum up time (MUT) and minimum down time (MDT)
Meaning of symbols:

\( I_i \) = Net income for unit i  
\( T \) = number of time intervals (planning horizon)  
\( \lambda_t \) = price of electric energy in hour t ($/MWh)  
\( \mu_t \) = price of reserve power in hour t  
\( p_{i,t} \) = unit i’s production in hour t  
\( P_{i,\text{max}} \) = maximum output of unit i  
\( FC_i \) = the fuel cost function of unit i  
\( SC_i \) = start-up costs

The states in the DP is whether the unit is running or not, and for how long. The number of up states is given by MUT, and the number of down states is the maximum of MDT and the cooltime of the unit. The cooltime of the unit is when the start-up costs no longer are increasing. With exponential start-up costs, see (2.6), cooltime is set equal to 4 times the time constant \( \tau_i \) in the exponential expression (\( e^{-4} = 0.018 \) which gives reasonable accuracy).

Each hour is a stage at which a decision has to be made.

The return function, \( NI(t) \), is the net income in a given hour t (stage). \( NI = 0 \) if the unit is not running, and if running:

\[
NI_t = \max_{P_{i,\text{min}} \leq p_{i,t} \leq P_{i,\text{max}}} \{ \lambda_t \cdot p_{i,t} + \mu_t \cdot (P_{i,\text{max}} - p_{i,t}) - FC_i(p_{i,t}) \} \quad (A.2)
\]

The maximization of (A.2) is trivial when fuel costs are given by (2.5). First optimal production without regard of min. and max. production, \( P_{i,t}^* \), is found:

\[
\frac{\partial NI_t}{\partial p_{i,t}} = \lambda_t - \mu_t - b_i - 2a_i p_{i,t} = 0 \quad (A.3)
\]

\[
P_{i,t}^* = \frac{\lambda_t - \mu_t - b_i}{2a_i} \quad (A.4)
\]

With regard to min. and max. production the optimal production, \( P_{i,t}^{**} \), is found
\[
\begin{align*}
    P_{i,t}^* < P_{i,t}^{\text{min}} & \Rightarrow P_{i,t}^{**} = P_{i,t}^{\text{min}} \\
    P_{i,t}^* > P_{i,t}^{\text{max}} & \Rightarrow P_{i,t}^{**} = P_{i,t}^{\text{max}} \\
    P_{i,t}^{\text{min}} \leq P_{i,t}^* \leq P_{i,t}^{\text{max}} & \Rightarrow P_{i,t}^{**} = P_{i,t}^*
\end{align*}
\] (A.5)

\(N_{I0} = 0\) for all states, income in hour 0 is not to be considered.

The transition cost is equal to the start-up cost when start-up occurs, and zero if no start-up occurs.

Figure A.1 shows the decision tree for a unit with \(MDT = MUT = 2\) hours and start-up cost dependent on downtime (start-up cost = \(C_1\) if \(t_{\text{down}} = 2\) hours and start-up cost = \(C_2\) if \(t_{\text{down}} > 2\)). If the unit has been down for \(MDT\) or more, it has to decide whether to start up or not, and if it has been up for \(MUT\) or more it can continue running or shut down.

The rectangles show the state of the unit, and the numbers has the following meaning:

-1: the unit has been down for 1 hour
-2: the unit has been down for 2 hours

Fig. A.1. Decision tree for a T-stage optimization problem. Unit with \(MDT = MUT = 2\) and time dependent start cost.
A.1 Backward Dynamic Programming applied to single unit optimization

-3: the unit has been down for 3 hours or more
+1: the unit has been up for 1 hour
+2: the unit has been up for 2 hours or more

For each node in the tree, the optimal accumulated net income is calculated by backward DP. The accumulated net income, ANI, for a node represents the optimal net income by starting in that node and following the optimal route in the decision tree to the end stage \((t=T)\).

Recursive relationship:

\[
\begin{align*}
ANI_{j,T} &= NI_{j,T} \quad j \in J \\
ANI_{j,t} &= \max_{i \in J} \{ ANI_{i,t+1} - TRC_{j,i} + NI_{j,t} \} \quad t = T-1, \ldots, 1, 0 \quad j \in J \quad (A.7)
\end{align*}
\]

Meaning of symbols:

\begin{itemize}
  \item \(ANI_{j,t}\) = Accumulated net income in state \(j\) at stage \(t\)
  \item \(TRC_{j,i}\) = Transition cost going from state \(j\) to state \(i\) (= Start cost if \(j\) is down state and \(i\) is up state, = 0 otherwise)
  \item \(NI_{j,t}\) = Net income in state \(j\) at stage \(t\). Equals \(NI(t)\) if \(j\) is an “Up state” (+1,+2, ...), zero if \(j\) is a “Down state” (-1, -2, ...)
  \item \(J\) = Set of all possible states (which is the same at all stages)
      \(\{-3,-2,-1,+1,+2\}\) for fig. A.1
\end{itemize}

The optimal values for different initial states found by the DP is used to calculate \(I_{\text{diff},i}\) which is needed in the bid calculation \((4.6)\). By using a backward DP and including a stage for \(t=0\), optimal solutions for all initial states are calculated (of which two are needed in the bid calculation).

Example:
The unit in fig. A.1 has been running for more than MUT, and is calculating its bid for hour 0. In hour 0 it has two options: Either continue running (which is represented by \{+2\} in hour 0 \((t=0)\) in fig. A.1), or shut down \((-1\) in hour 0 in fig. A.1). The difference in future profit between these two options can then be calculated:

\[
I_{\text{diff},i} = ANI_{+2,0} - ANI_{-1,0} \quad (A.8)
\]
$I_{\text{diff},i}$ is then used in calculating “MinPrice” in (4.6).
Appendix B

Lagrangian Relaxation

For benchmarking of the computer program that has been developed (SimCom), it is possible to calculate the dual value of the Lagrangian Relaxation method using the clearing prices from SimCom as the lagrangian multipliers. This dual value represent a lower value of the optimization problem. For this reason, the lagrangian relaxation dual function is derived here.

B.1 Lagrangian Relaxation (LR)

See [12] and [17] for details.

The optimization problem is stated in section 2.5. The first step of the Lagrangian Relaxation method is to include the global restrictions (2.2) and (2.3) in the object function (2.1):

\[
\begin{align*}
\min_{\mathbb{u}, \mathbb{p}} L &= \sum_{t=1}^{T} \sum_{i=1}^{N} (FC_{i,t} + SC_{i,t}) + \sum_{t=1}^{T} \lambda_t \left( P_{D,t} - \sum_{i=1}^{N} p_{i,t} \right) + \sum_{t=1}^{T} \mu_t \left( R_t - \sum_{i=1}^{N} (P_{i,t}^{\max} u_{i,t} - p_{i,t}) \right) \\
\end{align*}
\]

(B.1)

Local restrictions (unit restrictions) remains as before.

\( \mathbb{u} \) is the set of all \( u_{i,t} \) (i.e. for all \( i \) and all \( t \))

\( \mathbb{p} \) is the set of all \( p_{i,t} \)

Since (2.2) is an equality constraint, each \( \lambda_t \) is a free variable, and since (2.3) is an inequality constraint each \( \mu_t \) must be non-negative. \( \mathbb{\lambda} \) and \( \mathbb{\mu} \) are called the lagrangian multipliers (\( \mathbb{\lambda} \) is the set of all \( \lambda_t \) and \( \mathbb{\mu} \) is the set of all \( \mu_t \)).

Since (B.1) is a relaxation of the original problem, the solution of (B.1) for any given set
of $\lambda$ and $\mu$, will be a lower limit to the original minimization problem.

Equation (B.1) is separable, and can be divided into a subproblem for each unit:

$$D = \min_{u, p} L = \sum_{i=1}^{N} K_i(u, p) + \sum_{t=1}^{T} (\lambda_t \cdot P_{D,t} + \mu_t \cdot R_t) \tag{B.2}$$

rewritten to:

$$D = \sum_{i=1}^{N} \min_{u_i, p_i} \{K_i(u, p)\} + \sum_{t=1}^{T} (\lambda_t \cdot P_{D,t} + \mu_t \cdot R_t) \tag{B.3}$$

where

$$K_i(u, p) = \sum_{t=1}^{T} FC(p_{i,t}) + \sum_{t=1}^{T} SC_{i,t} - \sum_{t=1}^{T} (\lambda_{t} \cdot p_{i,t} + \mu_{t} \cdot u_{i,t} \cdot (P_{i}^{max} - p_{i,t})) \tag{B.4}$$

(B.3) is called the dual function, since maximizing this function over $\lambda$ and $\mu$ subject to the local constraints gives a lower limit to the original problem.

Each subproblem is small, and can easily be solved by Dynamic Programming. The value of the dual function can therefore easily be calculated when $\lambda$ and $\mu$ are fixed.

The dual problem is:

$$\max_{\lambda, \mu} D = \max_{\lambda, \mu} \left( \min_{u, p} L \right) \tag{B.5}$$

subject to the local constraints.

In LR, the commitment plan $(u)$ generated from the solution of each subproblem is used to generate a primal feasible solution. If the commitment plan cannot be used directly, some heuristic rules are used to generate a feasible solution.

The multipliers are updated in each iteration of the LR. Several methods exists for updat-
ing the multipliers.

The value of the primal function is always greater than or equal to the dual value (weak duality). The difference between the two functions yields the duality gap. The duality gap provides a measure of the near-optimality of the solution.

Examples of the Lagrangian Relaxation method to solve the short term scheduling problem are shown in [18] and [21].

For an overview of optimization algorithms for the scheduling problem see [28] and [29].
Appendix C

Case data

Test cases 1 & 2 are both from [23]. Test case 3 is provided by Professor Arne Johannesen of SINTEF Energy Research.

C.1 Test case 1

Table C.1 Data for test case 1. Exponential start-up cost function, see (2.6). IC = initial condition (number of hours up if positive value, else number of hours down)

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<th>P\text{min} (MW)</th>
<th>MDT (h)</th>
<th>MUT (h)</th>
<th>IC (h)</th>
<th>a ($/MW)</th>
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## C.2 Test case 2

Table C.3 The unit characteristics and cost coefficients.

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Table C.3 The unit characteristics and cost coefficients. Exponential start-up cost function, see (2.6). IC = initial condition (number of hours up if positive value, else number of hours down)

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Table C.3 The unit characteristics and cost coefficients.
Exponential start-up cost function, see (2.6). IC = initial condition (number of hours up if positive value, else number of hours down)

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<th>Hours 13-18</th>
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Table C.4 The load demand in MW (110 unit test system)
(Reserve requirement is 700 MW in all hours)

<table>
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<tr>
<td>Hours 7-12</td>
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<tr>
<td>Hours 13-18</td>
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<tr>
<td>Hours 19-24</td>
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C.3 Test case 3 (9 units, 168 hours)

Table C.6 Load demand in MW. Simulation period is 5 weekdays followed by 2 weekend days. Reserve requirement is 10% of load in all hours.

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<th>Hour</th>
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Table C.7 Unit data for test case 3. The units have no minimum up-time or minimum down-time, and start-up cost is constant. (IC = initial condition)

<table>
<thead>
<tr>
<th>Unit</th>
<th>P&lt;sup&gt;max&lt;/sup&gt;</th>
<th>P&lt;sup&gt;min&lt;/sup&gt;</th>
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<th>a</th>
<th>b</th>
<th>c</th>
<th>Startcost</th>
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Appendix D

Examples

This appendix contains numeric examples of quantity determination and bid calculation.

D.1 Example of calculation of a units production quantity

This procedure is described in section 4.3.5.

Consider a unit which has bid the price 20 for all production-quantities from $P_{\text{min}}$ to $P_{\text{max}}$. Its marginal cost function is $MC(p) = 5 + 0.5p$. $P_{\text{min}} = 10$ and $P_{\text{max}} = 20$. Market clearing prices are $\lambda_c = 19.6$, $\mu_c = 7.6$.

First, the quantity to produce if the unit is set to run ($p_r$) is found from the marginal cost curve:

$$\lambda_c - \mu_c = 5 + 0.5p_r \Rightarrow p_r = (19.6 - 7.6 - 5) / 0.5 = 14.0$$

Then it is checked whether the unit will run with the given prices. For the quantity of 14.0, the bid-price is 20, but since this is on a flat segment of the bid, $p_b$ is set to the end of that segment; $p_b = P_{\text{max}} = 20$ and the price asked is $\lambda_b = 20$. The required income is given by (4.8):

$$I_{\text{req}} = 20 \cdot \lambda_c - (5 + 0.5 \cdot 14.0 + 5 + 0.5 \cdot 20) \cdot (20 - 14.0) / 2 = 319.0$$

$$I_{\text{req}} = 20 \cdot 20 - (5 + 0.5 \cdot 14.0 + 5 + 0.5 \cdot 20) \cdot (20 - 14.0) / 2 = 319.0$$

Income $= p_r \lambda_c + (P_{\text{max}} - p_r) \mu_c = 14.0 \cdot 19.6 + (20 - 14.0) \cdot 7.6 = 320.0$

Income $> I_{\text{req}} \Rightarrow p^* = p_r = 14.0$
The unit will produce 14.0 at these clearing prices.

### D.2 Example of Bid Calculation

Here is an example of how a thermal generator calculates its bid for the next hour (hour 1). It as a price forecast for the next 24 hour (the price forecast for hour 1 is not needed and is not presented in the table below).

The unit in question shall calculate its bid for hour 1. Its price forecast is given by table D.1 ($\lambda$ and $\mu$). The unit has minimum up time of 6 hours, minimum down time of 6 hours, start-up cost of $1800$. Fuel cost function: $FC(p) = 450+19.7p+0.004p^2$, minimum production of 25 MW and maximum production of 162 MW. The unit has been down for more than 6 hours

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<th>$\mu$</th>
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<th>Prod. cost</th>
<th>Net income</th>
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<td>945.0</td>
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<td>25.0</td>
<td>487.5</td>
<td>945.0</td>
<td>-457.5</td>
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</table>
Table D.1 shows what the unit will produce in each hour if it is set to run. The production level is given by the marginal cost function and the energy and power prices. The table shows the net income in each hour at the calculated production levels.

Net income in each hour is also shown graphically in fig. D.1.

If the unit is set to run in hour 1, the optimal commitment plan for the unit will be to run also in hours 2-21 (with the prices in table D.1). This gives an income of $12,006.6.

If the unit is set to be down in hour 1, the optimal commitment plan is to run in the hours from 6 to 21, and not run in hours 2-5 and 22-24. This gives an income of $10,969.8.

Both optimal commitment plans are found by running a backwards Dynamic Programming (DP) routine once (see appendix A).

Then the impact on future income from running vs. not running in hour 1 can be calculated:

\[ I_{\text{diff}} = 12,006.6 - 10,969.8 = 1,036.8 \]

Then the minimum energy price required (for \( \mu = 0 \)) can be calculated from (4.6):
MinPrice_i = \min_{p_i \leq p_i \leq p_i} \frac{(a_i + b_i p_i + c_i p_i^2 + SC_i - I_{diff,i})}{p_i} \quad (D.2)

MinPrice_i = \min_{25 \leq p_i \leq 162} \frac{(450 + 19.7 p_i + 0.004 p_i^2 + 1800 - 1036.8)}{p_i} \quad (D.3)

Solving this gives: MinPrice_i = 27.84
Appendix E

More results from simulations

This appendix contains some of the same results that are shown as figures in chapter 6, but here they are shown in tabular form.

Table E.1. Schedule cost (in $1000) in each iteration of SimCom for different initial price forecasts (same price in all hours), for test case 2.

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</table>

| Best      | 3812.2 | 3812.2 | 3811.3 | 3810.9 | 3810.9 | 3810.9 |

% from best 0.036 % 0.036 % 0.011 %
Table E.2. Schedule cost in the first iterations of SimCom for different initial price forecast. The price forecast equals the price forecast giving the best solution multiplied by the scaling factor

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Table E.3. Price forecast and resulting prices for test case 2 in the iteration that gives the best solution

<table>
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<tr>
<th>Hour</th>
<th>Forecasted energy price</th>
<th>Resulting energy price</th>
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<th>Resulting reserve price</th>
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