ABSTRACT
Transformer models have a variety of applications and different requirements make use of different principles due to various demands regarding frequency range, needed degree of accuracy, application and so on. The history of transformer modelling is reviewed from the “early beginning” until today describing the main lines and principles used. The different challenges attended with transformer modelling are elucidated. Comparison of some of the principal articles is made in table 1 (appendix 1).

INTRODUCTION
The first transformer was built and patented in 1885 [1], with a ratio of 120 to 72 Volts (40 Hz) supplying 30 lamps (1400 VA). A common way of rating electrical networks at that time, was principally based on the number of lamps the network was capable of supplying. The development of the transformer during the first century was based mainly on experience rather than science. At the beginning of the last century it became clear that the response of transformers to lightning and switching surges was entirely different from the normal performance at operational frequencies, and the need for more detailed modelling and knowledge arose. The transformer is a very complex component to model and there are details that are not fully understood even today.

James Clerk Maxwell [2] died without any public honours, but is in modern science ranked with Newton and Einstein for among others the concept of electromagnetic radiation and his field equations (1873). These equations were hardly used due to difficult analytical solutions for practical geometries, until the development of numerical algorithms/digital computers. Maxwell’s equations are widely used nowadays, also in transformer modelling. They can be used to describe nearly all kinds of macroscopic phenomena covering both stationary and transient/high frequency fields, and forms the basis for electromagnetic modelling.

During the last 50 years the need for detailed transformer models have been the driving force of transformer research. The manufacturers needed a tool to design cost efficient and reliable transformers. The utilities needed transformer models to simulate transformer interaction and backfeed during faults in their networks, and also as a tool for insulation coordination. Lately transformer models have found new applications such as in the author's doctoral work: Sensitivity analysis of FRA as a diagnostic method.
THE EARLY AGES

The first era of transformer modelling is well described by Abetti’s bibliography [3-5] and historical survey [6]. The second era, from the early fifties until today, is mainly influenced by the use of digital computers and is treated in the following. Abetti’s bibliography is ranging from 1902 to the publication date of the last supplement [5] in 1964. His bibliographies contain 1265 references regarding transformer modelling and he collected them over a period of 10 years. Because of its importance and lack of credits from recent articles, a very brief summary is given. Abetti also contributed [7], [8] to the topics of surge response and transformer modelling and his work represents a milestone in transformer modelling.

In 1902, Thomas [9] discovered that the voltage distribution in transformer winding exposed to surges with steep fronts is far from linear, resulting in very large voltage gradients in limited parts of the winding. Some years later Steinmetz, when discussing a paper by Jackson [10], indicated that a winding resembles more like a capacitance than an inductance at high frequencies. This led to the discussion of reinforced end-turn insulation, and later shielding and “inter-leaving” to control the electrostatic voltage distribution. The initial (electrostatic) voltage distribution was recognized to follow a certain hyperbolic pattern. For the subsequent oscillations, the solution was attempted in many ways, which at the time where classified according to the following three methods:

1. Treating the winding as a transmission line, was first done by Rudenberg [11], [12].
2. Representing the winding by means of a ladder network (mesh) of concentrated parameters was first done by Weed [13]. He neglected the mutual inductances. (Included by Wagner [14], [15] but only between adjacent elements).
3. An equivalent circuit of distributed inductive and capacitive parameters were suggested by Wagner [16], still with mutual inductances only between adjacent elements. He solved the differential equations for the oscillations by applying as boundary conditions the initial and final distribution.

These 3 representations have subsequently been widely used and further developed.

During the 1920’s attempts were made to get more precise solutions by including all self- and mutual inductances. This attempt led to highly mathematical and unsatisfactory results, and found no practical applications until 30-40 years later (using computers).
**INTRODUCTION TO MODELLING PRINCIPLES GENERALLY USED TODAY:**

In 1919 Blume and Boyajian [17] introduced the concept of leakage inductance of the winding and methods for determining leakage inductances, which describes the difference between the self- and mutual inductance of two filaments directly. Rosa [18] made one of the important contributions in calculation of self-inductances in 1906 and was later extended by Roth [19] and then Grover [20]. Another approach of transformer modelling is the principle of duality, which was introduced by Cherry [21]. Duality means representing magnetic circuits with electrical equivalents. In the 50’s and 60’s manufacturers used reduced scale models [7] to improve their designs, but because it is a time-consuming and expensive way of modelling, they searched for new methods. Measurements can also be used to identify high frequency characteristics of transformers [22]. The last group of modelling methods is based on electromagnetic fields and have developed through the last 40 years due to the development of computers.

**THE AGE OF COMPUTERS:**

The introduction of computers enabled transformer engineers to solve their theoretical problems with higher accuracy and less simplifications than earlier. Today all methods of transformer modelling is based on computers, some demanding severe resources, others not. The main problem in the beginning of this era was the computation of the inductances. Most of the capacitances could be calculated analytically, but both the inductances and losses are frequency dependent and need advanced numerical techniques to be calculated with sufficient accuracy.

**Inductance Calculation**

The main streams of inductance calculation for analysis and design of transformers can be classified as:

- **Modelling based on self and mutual inductances.** The first analytical, computer-based attempt following this approach was presented by Rabins [23] followed by many others such as Fergestad and Henriksen [24], [25] (mainly based on Olaussen [26]) and recently continued by Wilcox et al. [27], [28]. There are very accurate formulas available for the calculation of self and mutual inductances for the windings, sections, or turns of transformers. However, because of the presence of the iron core, the numerical values of the self and mutual inductances are very close and result in ill-conditioned equations.

- **Modelling based on leakage inductance.** This approach was, as mentioned earlier, initiated by Blume [17] and improved by McWirther et al. [29] and Shipley et al. [30]. Brandwajn et al. [31] presented the three-phase multi-winding generalization. Dugan et al. [32] used the same technique for modelling multi-section transformers. These models represent the leakage inductance of the transformer adequately (i.e. load or short circuit conditions), but the iron core is not properly included.
Modelling based on the principle of duality. As mentioned earlier this approach was introduced by Cherry [21]. The iron core can be modelled accurately. However, models based only on this approach have the inconvenience that the leakage inductances are not correctly represented (they are directly derived from the leakage flux neglecting the thickness of the windings). Edelmann [33] and Krähenbühl et al. [34] corrected this inaccuracy (assuming that the magnetic field is axial). Arturi [35] used this approach in the modelling of highly saturated conditions.

Modelling based on measurements (black-box modelling). A great number of high frequency transformer models have been derived from measurements; see for instance [22], [36], [37]. Tests are made for the determination of the model parameters in the frequency domain or time domain. Models obtained from measurements have the drawback that their performance can only be guaranteed for the tested transformers. Although some general trends can be inferred from the tests, according to design, size, manufacturer, etc., accurate predictions for non-tested transformers cannot be assured. The main methods used for analysing black-box measurements are:

- Modal analysis: Wilcox et al. [38]-[40] and Glaninger [41], [42].
- Pole/zero-representation: Soysal et al. [43].

Analysis based on electromagnetic fields. Designers of large transformers use electromagnetic field approaches for the calculation of the design parameters. The technique of finite elements (FEM) is the most accepted numerical solution for field problems [44]. There are, however, other techniques available; see for instance [45]. There is general agreement that three-dimensional field analyses are necessary in the design process. These methods are in most cases impractical for the calculation of transients since they give expensive simulations, at least for 3D-calculations. Recently clusters of Linux-based PC’s have shown promising results for demanding calculations and will contribute to make such calculations considerably cheaper in near future. Some applications have been developed for transients, and currently the author is preparing a stay at EdF in France, where he is going to use their FEM-application [46] to model his test-objects and compare with measurements.

Transmission Line Modelling: The winding is treated as a multiconductor transmission-line. This approach can only be applied on homogenous windings according to Al-Khayat [47].

Combinations: Several publications have combined the methods above. The method of DeLeon/Semlyen [48], [49] was derived from a combination of leakage inductances and principle of duality. Gharehpetian et al. [50] combined the principle of self- and mutual inductances with the black-box method. This extended the validity of the model from a few hundred kHz to a few MHz.
The methods listed above concerns mainly the inductances in the transformer model. Other important elements in a high frequency transformer model are the capacitances and the losses.

**Capacitance Calculation**

The model capacitances can be calculated either by using traditional analytical methods or by using computer methods such as the Finite Element Method where the material parameters and the geometry are important [51]. Shunt- or parallel capacitances (capacitance between windings or from winding to ground) can be calculated with simplified geometrical formulas or on a semi-empirical basis (additional capacitance to ground are introduced by connections, bushings, static plates and tap-changers. The series capacitance is the capacitance between different turns of the same winding and is a determinant for the electrostatic voltage distribution. Several contributions to the calculation of this parameter are made: (Stein [52], Okuyama [53] and Ambrozie [54], [55]). Special consideration must be taken when calculating the series capacitance of interleaved coils or coils with in-wound shields (See: Ambrozie [55], [56], Pedersen [57], De [58], Moreau et al. [59], Del Vecchio et al. [60] and finally Seitlinger [61])

**Losses**

The losses in a detailed model are indispensable, particularly when internal stresses are evaluated in the design-stage of a transformer. Without the losses implemented, the stresses will be higher in the model than in reality and the design will become unnecessary cost-consuming and then less competitive, as stated by Mombello et al. [62].

The different loss-mechanisms in a transformer are:

- Series resistance in windings (DC-resistance)
- Frequency dependent losses in the conductors occur as eddy currents due to time-varying external magnetic field. The eddy currents cause an increase of the losses and a reduction in the net amount of magnetic flux. This is translated into an increase of the resistance and a reduction of the inductance of the equivalent impedance representing the winding.
  - Skin effect: change in current density distribution due to the current in the conductor itself, as shown by DeLeon et al. [63], [64].
  - Proximity effect: eddy currents due to the external magnetic field generated from current in the other conductors, see . [63]-[73].
- Eddy Currents in core laminations (core reaction) due to magnetic field in the core. The eddy currents have a frequency dependent penetration depth, shown by Ferreira [74], [75].
- Dielectric losses within the insulation (both series- and shunt conductances) due to conductivity and different polarization mechanisms. Modelling may be based on measurements done by Buckow [76].
MODEL APPLICATIONS

There are several different applications for the different transformer models:

- Determining impulse-overvoltages in windings, during both design-stage and when coordinating isolation-levels. The impulse strength is checked during factory acceptance tests. Detailed methods are necessary, see; McWirther et al. [29], Dent et al. [77] and Okuyama [78].
- Understanding measurements and propagation of signals in windings due to partial discharges (locating partial discharges). Such considerations are made by Dong [79].
- Analysing resonances in power networks and transformers.
  - *Lightning overvoltages*: These usually correspond to fast wave fronts (Miki et al. [80]).
  - *Switching overvoltages*: These correspond to slower wave fronts, and can be generated by switching of lines, transformers, reactors and faults (Degeneff et al. [81]).
- Understanding frequency response measurements when applied in diagnosis and condition assessment. According to Rahimpour et al. [82]-[84], the most suitable in this application, is the method of self- and mutual inductances using air core theory (see; [85] & [86])

SUMMARY

The history of high frequency transformer modelling covers a whole century of development and method-evolution, and reveals that there still are unsolved problems within transformer engineering. Different methods of modelling are listed and discussed. Articles with significant contributions regarding calculations are compared in table 1, appendix 1 and are considered being excellent starting points for high frequency transformer modelling. The choice of method is based on the application of the model. The remaining challenge in transformer modelling today is the connection between low frequency- and high frequency models. In this mid-frequency area, the non-linear effects (saturation and hysteresis) must be considered (see for example Furgal [87] and White [88])
REFERENCES


## APPENDIX 1: DIFFERENT SUBJECTS TREATED IN RFC’S:

| References: | [24] | [25] | [28] | [36] | [47] | [52] | [56] | [57] | [58] | [60] | [67] | [77] | [78] | [80] | [81] | [82] | [83] | [84] | [85] | [86] | [87] | [88] |
|-------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| **Circuit Analysis:** |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Continuous | X    |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Subdivided | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    |
| **Field Analysis:** |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Analytical Techniques | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    |
| Numerical Techniques |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Lumped-element model | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    |
| Multiconductor TL-model |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Self- and Mutual Inductances | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    |
| Leakage inductances |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Eddy Currents – Winding | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    |
| Eddy Currents - Core/Structural |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Hysteresis | X    |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Saturation | X    |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Dielectric losses |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Frequency dependent losses | (X)  | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    |
| Formulas for resistance of a winding |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Formulas for inductance (air core) | X    |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Formulas for inductance (iron core) | (X)  | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    |
| Formulas for capacitance |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Considers ac resistance | (X)  | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    | X    |
| Considers ac inductance |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Stacked Core Transformer | X    | X    |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Double-disc winding | (X)  | X    | X    |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Disc-winding |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Layer winding | X    |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Interleaved layer winding |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Interleaved disc-winding |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Interl. disc-wind. with inwound shields | X    | X    | X    |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Helical winding |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Uniform windings |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Non-uniform windings |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Time-domain solution | X    | X    |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Frequency-domain solution | X    | X    | (X)  |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| Matrix methods shown | X    | X    | X    |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| **(X)** |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |