Abstract—The choice of the most suitable control strategy for Wave Energy Converters (WECs) is often evaluated with reference to the sinusoidal assumption for incident waves. Under this hypothesis, linear techniques for the control of the extracted power, as passive loading and optimum control are well-known and widely analyzed. It can be shown, however, how their performances are fundamentally different when irregular waves are considered and the theoretical superiority of optimum control is questionable under real wave conditions. Moreover, the global optimization of WECs requires a rational design of the power electronics equipment. This requires the analysis of the instantaneous extracted power in addition to the average one. In this paper the impact of irregular waves on the power extraction when using different control techniques is analyzed in the case of a point absorber in heave. It is also shown how a convenient trade-off between high average power extraction and limited power electronics overrating can be obtained by applying simple power saturation techniques. Moreover, the impact of power conversion efficiency on the control strategy is analyzed.

I. INTRODUCTION

Among renewable energies the exploitation of ocean waves is certainly one of the most recent. In spite of its consistent potential [1], due to high availability and stability of the source and low environmental impact, a single leading technology has not yet been established and different concepts are being studied and tested worldwide. Among them, a promising solution is represented by point absorbers, i.e. floating buoys having small dimensions compared to the wavelength of incident sea waves. The principle of the power extraction lies in creating some kind of destructive interference between the incident wave and the oscillation of the buoy. This requires an active control of the device that needs to be tuned according to the sea state. Thus, the control of the motion is one of the crucial aspects to improve the power performance of Wave Energy Converters and many scientific contributions have been devoted to this topic [2-4]. The most of them focus on linear control techniques that are quite easy to implement and have been systematically studied in order to maximize the power extraction. It is worth noting that a large number of these studies are developed under the ideal assumption of sinusoidal incident waves [3-4]. It will be shown in the following, however, that this can lead to mislead the conclusions when real waves are considered. On the other hand, some studies on practical techniques to improve power performance have also been proposed, but being limited to few very specific applications, without necessarily claim to generality [5-7].

Moreover, when dealing with WEC performance analysis, the attention is usually focused mainly on mechanical and hydrodynamic aspects, while the issues related to the sizing and efficiency of electric and electronic equipment are often neglected or oversimplified. The goal of this paper is at first to underline the impact of irregular waves on the performance of linear control techniques. Following, some considerations on the relationship between the choice of the control strategy and the rating of power electronic equipment are presented. Consequently the usefulness of simple power saturation techniques in order to maximize the average extracted power, while avoiding excessive over ratings of the converter, is proposed and exemplified through computer simulations. The attention is then focused on the effects of a non ideal power conversion efficiency and its influence on the choice of the best control strategy is also shown. Finally, a complete model of the system, including the electrical machine and power electronic interface has been developed in Matlab/Simulink; hence the proposed control solutions have been tested, confirming the validity of the previous analyses.

II. MODEL OF THE WAVE ENERGY CONVERTER

A. Hydrodynamic model

The following analysis is focused on a spherical point absorber in heave [8], i.e. a single degree of freedom device as the one schematically depicted in fig. 1, which is directly connected to the electric Power Take Off (PTO) without any intermediate hydraulic or pneumatic stage.

Under the assumption of plane progressive waves propagating in an infinite water depth and if small motion is assumed, linear theory can be applied and the hydrodynamic diffraction model can be used to represent the interaction between the buoy and the waves [9].
A.1 Frequency-domain model
If sinusoidal incident waves (regular waves) are considered, the system behavior can be described in the frequency domain as [9]:

\[
-\omega^2 (M + a(\omega)) \ddot{X} + j \omega B(\omega) \dot{X} + K \dot{X} = F_E + F_L \tag{1.a}
\]

where \( \omega \) is the angular frequency of the incident wave, \( X \) is the buoy position, and "+" denotes complex quantities. \( M \) is the mass of the device (including also the contribution of the generator inertia \( J_m \), scaled by the gear ratio, \( n \)) and \( a \) is the added mass at the considered frequency. Added mass takes into account the water mass involved in the device movement and depends on the radiation force caused by device oscillation. \( B(\omega) \) is the mechanical damping (also including the radiation resistance \( b \), which is frequency dependent, too); \( K \) is the hydrodynamic stiffness. Moreover \( F_E \) is the excitation wave force and \( F_L \) is the force applied by the Power Take Off. Hydrodynamic parameters for the reference test case are listed in Tab. I.

A.2 Time-domain model
In order to analyze the WEC behavior in irregular waves and to cope with any non-linearity, a time domain model is required, as the following one based on the Cummins equation [10]:

\[
(M + a_\infty)\ddot{x}(t) + \int K_{\text{rad}}(t - \tau)\dot{x}(t)d\tau + Kx(t) = F_E(t) + F_L(t) \tag{1.b}
\]

In (1.b) \( a_\infty \) represents the value of added mass for \( t \to \infty \) and "+" is time derivation operation. Moreover \( K_{\text{rad}} \) is the Radiation Impulse Response Function (RIRF). It is worth noting that the radiation force acting on the buoy is non-causal.

B. Electric analogue of the WEC under sinusoidal conditions
The system described by (1.a) corresponds to a mass-spring-damper system and, in order to gain a better understanding of its behavior, it can be useful to introduce its electric analogue (Fig. 2), which is valid as long as regular waves are assumed. The excitation force, \( F_E \), corresponds to the sinusoidal voltage, \( e \), and the buoy velocity \( \dot{x} \) to the current, \( i \). Moreover, the device mass \( M \) is represented by the inductance \( L \), the spring stiffness, \( K \), by the inverse of the capacitance \( C \) and, finally, the mechanical damping, \( B \), by the resistance \( R \). Thus, the buoy, as a whole, is represented by the impedance \( Z \). Correspondingly, the force applied by the Power Take Off corresponds to the load voltage \( u_L \), whose value is related to the control parameters represented by load resistance \( R_L \) and load reactance \( X_L \), as will be better explained in Section III.

C. Model of the WEC for irregular waves tests
The time domain model of the WEC has been built from (1.b) and is reported in Fig. 3. The excitation force, \( F_E \), coming from the waves is considered as a system input and its derivation is described in detail in Section 3.C. The control force applied by the PTO is a system input as well. From the availability of the hydrodynamic coefficients of the point absorber described in [4], the corresponding radiation impulse response was obtained and the frequency response derived. An identification method (Matlab command: N4sid) was then applied to such frequency response in order to finally model the radiation force (convolution integral) as a 4th order state space system.
D. Model of the Power Take Off

The PTO is composed by an electric machine, controlled by a bidirectional switching converter. Field Oriented Control (FOC) is used to control the torque (i.e. the force) applied by the PTO. One of the goals of the following power performance analysis is to provide information about the sizing of the generator and the power converter.

III. CONTROL OF THE WAVE ENERGY CONVERTER

A. Linear control strategies

The simplest and most widespread control strategy for heaving buoys is passive loading, where the force exerted by the Power Take Off is proportional to the buoy velocity. Acting this way, only the amplitude of the buoy motion can be controlled. On the other hand, both the amplitude and the phase of the motion need to be controlled in order to absorb the absolute maximum average power. This is realized by implementing the optimum (reactive) control, where the applied force has a component which is proportional to the buoy acceleration (reactive component) in addition to the one proportional to the velocity (damping component). In this case the theoretical condition of resonance between the motion of the device and incident waves is achieved; however, a bidirectional power flow between the buoy and the PTO must be allowed.

B. Sinusoidal incident waves

The power performance of point absorbers is well-known if sinusoidal incident waves are considered [9] and it is here recalled with reference to the most consolidated control strategies for comparison purposes. Passive loading application corresponds to have a zero control reactive component, i.e. \(X_L=0\).

To obtain the maximum possible extraction of average power under this condition, the resistive component must be selected so that:

\[
R_L = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}
\]

On the other hand, application of optimum control implies:

\[
X_L = -(\omega L - \frac{1}{\omega C})
\]

\[
R_L = R
\]

leading to the resonance condition and, thus, to the absolute maximum of the average power extraction. The sinusoidal reference wave used in the following analysis has amplitude \(A=0.5\) m and period \(T=6\) s (Fig. 4.a).

C. Irregular incident waves

In order to evaluate the real performance of a WEC the analysis of its behavior when subject to irregular waves is mandatory. It is especially meaningful to develop some time-domain studies related to the power extraction. The stochastic character of real waves can be well understood if the superposition of infinite sinusoidal waves of infinitesimal height, random phase and different frequencies is considered. Several different models have been developed to provide a suitable description of a real sea. Most of them use few fundamental parameters that are considered representative of a specific sea state to build up the energy spectrum of the sea itself. In the following, the Bretschneider model [11] will be adopted, where the energy spectrum (Fig. 5.a) can be obtained as a function of the selected wave period and significant wave height of the sea state. From the energy spectrum the time domain expression of incident waves is derived and, if the geometry of the buoy is known, also the
excitation force it receives from the sea [12]; an example is shown in Fig. 5.b.

It is worth noting that, for the purpose of control strategies comparison, the energy period (also known as “true average period”) $T_e = 6$ s and the significant wave height $H_s = 1.41$ m that are used to build the energy spectrum for the following analyses, are selected according to a criterion of “equal energy period, $T_e$” (4.a) and “equal energy transport, $E$” (4.b) between the cases of sinusoidal and irregular waves [11]:

$$T_e = T$$

$$E_{\sin} = \frac{\rho g A^2}{2}$$

$$E_{\text{irr}} = \frac{\rho g H_s^2}{16}$$

$$E_{\sin} = E_{\text{irr}} \rightarrow H_s = 2\sqrt{2}A$$ \hspace{1cm} (4.b)

IV. ANALYSIS OF THE POWER PERFORMANCE WITH DIFFERENT CONTROL STRATEGIES

A. Impact of Control Strategies on the Converter Rating

In order to rationally choose the WEC control strategy, the rating of the electric and power electronics equipment must be taken into account. For such purpose, one interesting parameter is the instantaneous extracted power or equivalently the ratio, $k$, between peak and average power.

By simple analytical considerations it can be shown that under sinusoidal conditions ($T = 6$ s, $A = 0.5$ m) if passive loading is applied, the peak power always doubles the average one. In the presented case the average power that can be extracted is of 23.73 kW (Fig. 4.b). On the other hand, if optimum control is adopted under sinusoidal conditions, $k$ is largely varying depending on the specific device and incoming waves: in the considered example the adoption of optimum control leads to an average power of 56.22 kW and peak to average power ratio is equal to 4.54 (Fig. 4.c). Thus, under ideal conditions, optimum control enables the extraction of a higher average power compared to passive loading, at the expense of significantly higher power electronics rating.

It is then interesting to verify if the same trend applies in the case of irregular incident waves.

B. Power saturation effect

The analysis of the instantaneous trend of the power extracted in irregular waves shows that it is extremely fluctuating with sporadic very high peaks. If the conflicting need to maximize the average extracted power, while limiting excessive overrating of electronic equipment is also taken into account, it is useful to consider the effect of saturation on the instantaneous power, on the average power extraction. Such saturation is obtained by conveniently reducing the PTO applied force, so that the stated power limit is never exceeded. This effect is represented in a very simplified way, since more practically a torque limitation (possibly including some time-dependant overloading conditions) and a power converter current limitation are usually applied. All these provisions are here aggregated in the PTO power saturation.

C. Efficiency of the Power Conversion

When trying to quantify the useful extractable power from a WEC, another important consideration regards the efficiency of the whole power transfer. It must be considered
that electric machines and power converters have an efficiency that is affected by several factors, such as the loading conditions, the rotational speed, etc.; although a precise evaluation of the effect of efficiency is rather complex the simplified efficiency curve of Fig. 6a will be used as a function of the load factor, i.e. the ratio between the extracted power and the nominal (rated) one. As expected, the efficiency is strongly decreased when the extracted power is a small fraction of the nominal one. This has a different effect depending on the chosen control strategy.

Two different aspects must be taken into account, that are explained in the following, mainly with reference to the ideal sinusoidal case. The first and most important one deals with the fact that, if passive loading is applied, the power flow is unidirectional, while if reactive control is selected the power flow is bidirectional. This implies that, in the case of reversible power flow, a reduction of the extracted power needs to be taken into account in both directions (Fig. 6c), thus being a disadvantage for reactive control both in regular and in irregular waves.

Moreover, under certain conditions, the application of reactive control can lead the system to work in a “low load region” for a longer part of the period, thus contributing to disadvantage reactive control over passive loading. It’s better to point out, however, that this second effect strongly depends on the specific test case (i.e. peak over average extracted power) and it may become negligible in irregular waves. In the case exemplified by Figs 6.b-c the average power loss due to non-ideal efficiency results of 5% for passive loading and 12% for optimum control.

V. SIMULATION RESULTS

The previous analyses have been verified by Matlab/Simulink® simulations, according to the test conditions reported in Table I for both regular and irregular waves. Although targeted to a specific test case, the analyses can be extended to different system ratings and operating conditions.

A. Linear controls performance in regular and irregular waves

At first, sinusoidal ideal conditions were tested to be used as a reference case and to clearly emphasize the difference in the power performance when applying the different control strategies. As already mentioned, if the sinusoidal reference wave is assumed, passive loading leads to extract 23.73 kW, corresponding to only the 42% of the power that can be theoretically extracted by applying optimum control.

Thus, for the sinusoidal case, a clear advantage in using optimum control would be apparent. Then, in order to compare the power performance of passive loading and optimum control in irregular waves to the corresponding ones under ideal conditions, wave’s force profiles were created, in the form of profiles of 1000 seconds each (being the spectrum discretization frequency resolution $\Delta f = 0.001\text{Hz}$).

While the data about average power extraction are the same irrespective of the specific force profile that is applied (when all of them are derived from the same spectrum), the information about instantaneous power are related to the specific test case since phases at different frequencies are randomly generated. Thus the evaluation of the peak power (and, consequently, of power ratio) must be performed on a statistical base [10], considering the results coming, in our case, from the simulation of 80 different force profiles. The first set of simulations was aimed at the quantification of the power performance when passive loading is used. In this case the value of the damping component applied by the PTO is fixed and equal to the ideal damping (2) for a sinusoidal wave whose period equals the modal period [11] of the energy spectrum. The average power can be quantified into 18.38 kW (Fig. 7). When compared to the corresponding sinusoidal case, it shows how, due to irregular waves, a reduction of almost 1/4 of the average power is to be expected.

The same kind of test was developed while applying optimum control. Also in this case the reactive and damping coefficient are selected as the ones maximizing power absorption under sinusoidal conditions when a sinusoidal incident wave having period corresponding to the modal
period of the selected energy spectrum is considered (3.a-b). The corresponding power absorption is reported in Fig. 8. The total average extracted power is now of 28.38 kW, which is even more significantly reduced with respect to the corresponding sinusoidal case. Then the statistical analysis on irregular waves shows that, when passive loading is applied, the peak to average power value is in the range 7.7-17.1 and a power peak of 314 kW can be reached. In the case of reactive control $k$ is between 25.2 and 58.3 and the peak power absorption (over the 80 tested cases) exceeds 1.6 MW.

Thus the irregular wave study clearly confirms the need for a consistent overrating of the electric and electronic devices.

B. Test on power saturation effect

The second part of the analysis is focused on the effects of power saturation, representing the maximum power that can be handled by the electronic equipment and it is performed with reference to a specific force profile.

Eight different saturation levels are chosen for both passive loading and optimum control and analyzed with respect to both irregular waves and corresponding sinusoidal waves. Results are reported in Tab. II. It can be observed that in irregular waves a saturation of the maximum instantaneous power of 62% (from $P_{\text{sat}} = 147.04$ kW to $P_{\text{sat}} = 55.14$ kW) leads to a loss of average power only of 3% for passive loading, while in the case of optimum control a corresponding reduction of 67% in the maximum power (from $P_{\text{sat}} = 1102.8$ kW to $P_{\text{sat}} = 367.6$ kW) results in a 19.5% loss of average power, thus meaning that a significant reduction in the power electronics rating can be achieved with limited average power drop. With reference to the optimum control it is also worth noting that, if an even smaller saturation of 43% (from $P_{\text{sat}} = 255.2$ kW to $P_{\text{sat}} = 147.04$ kW) is applied under sinusoidal conditions, it leads to an average power loss of 63%, thus meaning that the beneficial effect of a power saturation is specifically related to irregular waves and could be not appreciated if only sinusoidal waves are considered. From the data of passive loading applied in irregular waves it can be also noted how the adoption of a non-linear control (i.e. saturation) can lead to the absorption of a higher average power than the correspondent (unconstrained) linear technique, under certain conditions, as already reported in literature [4]. Finally it is interesting to note the fact that, if the same limit for peak power is assumed for both passive loading and optimum control (e.g. $P_{\text{sat}} = 73.52$ kW), the implementation of passive loading results much more convenient than reactive control, if irregular waves are considered.

C. Tests on non-ideal efficiency effect

The goal of the following set of simulations is to evaluate the effect of a non-ideal efficiency of electric and electronic devices (fig. 6.a) on the power conversion and its specific impact on the different control strategies.

In Fig. 9, the average power extraction as a function of the power saturation level is reported for both ideal and non-ideal conversion efficiency, in the case of passive loading and optimum control.

The first important thing to underline is that, irrespective of the control technique that is applied, the effect of non-ideal conversion efficiency is of determining a point of maximum extraction efficiency of the power conversion, for passive and optimum control.

In Fig. 10, the average power extracted with non-unity efficiency as a function of the reactive component (evaluated as a percentage of the value used in optimum control). Resistive component is the one used in passive loading, while in the case of optimum control, it is also noted how the adoption of a non-linear control (i.e. saturation) can lead to the absorption of a higher average power than the correspondent (unconstrained) linear technique, under certain conditions, as already reported in literature [4]. Finally it is interesting to note the fact that, if the same limit for peak power is assumed for both passive loading and optimum control (e.g. $P_{\text{sat}} = 73.52$ kW), the implementation of passive loading results much more convenient than reactive control, if irregular waves are considered.

The maximum power extraction obtained from passive loading ($P_{\text{avg}} = 16.7$ kW) is more than
the maximum power extraction that can be reached by optimum control even in its most favorable condition ($P_{\text{avg}} = 15$ kW). Moreover, the highest power extraction in the passive loading case can be reached at expense of a consistently reduced power electronic equipment ($P_{\text{sat}} = 75$ kW), compared to the optimum control option (having its maximum at $P_{\text{sat}} = 500$ kW). Thus, in the considered test case, optimization of the average power extraction can be obtained by designing electric and power electronics equipment whose power rating is around 100 kW and by choosing passive loading as the most convenient control strategy.

The proven superiority of passive loading over optimum control when non-ideal efficiency is taken into account, leads to wonder if an “intermediate control” solution can ensure a higher average power extraction than passive loading under the same conditions. A specific test was carried out by adopting a saturation level of 110 kW. The resistive component of the control was fixed as in the case of passive loading while a reactive component being an increasing fraction of the one prescribed by optimum control was added. Average extracted power was then evaluated as the mean value of several sets of simulations. As can be seen from Fig. 10, when a reactive component being 50% of the one adopted in optimum control is applied, an average power of 21.8 kW can be obtained, corresponding to a 23% increase with respect to pure passive loading. Consequently, an intermediate control solution including a suitable reactive component is advisable even under non-unity efficiency conditions and must be carefully optimized to improve the whole power performance.

D. Detailed system simulation

Based on the proposed analysis, detailed simulation model including the electrical drive and the grid-interface converter has been developed. For this specific example, the electric machine is an asynchronous generator with a nominal power of 110 kW and a rated speed of 1487 rpm, while other electrical parameters are reported in Table III. The machine model is implemented in the dq reference frame and the Field Oriented Control has been adopted, as shown in Fig. 11 [13]. Two-level Voltage Source Inverter (VSI) has been adopted both for the electrical drive and for the grid-connected converter with $f_{\text{sw}} = 8$ kHz. The second one behaves as an Active Front End (AFE), where an inner dq current control imposes the line currents to be in phase with the line voltages and an outer control regulates the dc-bus voltage. The control algorithm has been implemented in the dq rotating reference frame and the angle $\theta$, needed for the transformation, has been generated by a Phase-Looked-Loop (PLL). The control used both for the grid-connected VSI and for the electrical drive is based on well-established knowledge available in the literature. Main converter parameters are also listed in Table III. The PI regulators of the current loops of both the electrical drive and the AFE are tuned to achieve a bandwidth of 500 Hz with a phase margin of 60°. The PI regulating the DC link voltage is designed for a bandwidth of 20 Hz and a phase margin of 70° and a feedforward action is added in the dc bus voltage loop (i.e. a term proportional to the generated power is added at the output of the PI_Vdc) to reduce dc-link voltage fluctuation.

![Fig. 11 Block diagram of the detailed system simulated model, including main control components](image-url)
TABLE III – PARAMETERS OF THE FULL-SYSTEM SIMULATED

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit of measure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_n$</td>
<td>110</td>
<td>[kW]</td>
<td>Generator nominal power</td>
</tr>
<tr>
<td>$V_n$</td>
<td>400</td>
<td>[V]</td>
<td>Generator nominal voltage</td>
</tr>
<tr>
<td>$L_m$</td>
<td>10.4</td>
<td>[mH]</td>
<td>Generator mutual inductance</td>
</tr>
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<td>10.6</td>
<td>[mH]</td>
<td>Generator stator inductance</td>
</tr>
<tr>
<td>$L_r$</td>
<td>10.6</td>
<td>[mH]</td>
<td>Generator rotor inductance</td>
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<td>[mΩ]</td>
<td>Generator stator resistance</td>
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<td>[mΩ]</td>
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<td>EMI filter inductance</td>
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<td>DC link voltage</td>
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<td>[m]</td>
<td>Pignon radius</td>
</tr>
<tr>
<td>$J_m$</td>
<td>2.3</td>
<td>[kgm²]</td>
<td>Generator mechanical inertia</td>
</tr>
</tbody>
</table>

One example of the results obtained with the detailed model is reported in Fig. 12. Fig. 12a shows the irregular wave profile and Fig. 12b the torque applied by the electrical machine when passive loading is used. It can be observed that the DC voltage (Fig. 12.c) reflects the low frequency oscillations of sea waves and such fluctuation is present also in the grid current waveform (Fig. 12.d). In this specific example, the dc-link capacitor is not providing any energy smoothing and the power fluctuation generated by the PTO is directly injected into the grid. From Fig. 13 it can be noted that the current is injected into the grid with a unity power factor, as required.

The detailed model confirms the analysis previously reported in Fig. 9. As an example, using a power saturation equal to 110 kW, the average extracted power using the detailed model is 16.4 kW, as marked by the square shown in Fig. 9.

VI. CONCLUSIONS

This paper has analyzed the effect of irregular waves in the control and design of wave energy converters. At first the effect of irregular waves when two different linear control techniques are applied has been shown. Consequently, from the consideration of the highly varying power profiles, the convenience of adopting power saturation provisions to reduce the rating of power electronics equipment without excessive drop in the average power extraction has been proved by computer simulations. Finally the non-unity efficiency of the real electric power conversion has been considered in order analyze its different impact on the selected control techniques.

REFERENCES


Fig. 12. Sea waves profile (a), generator torque (b), DC voltage reference and DC actual voltage (c), current injected into the grid in phase a(d)

Fig. 13. Detail of the voltage and current of phase a at the grid section