Abstract — This paper presents how to analytically design a high-torque three-phase flux-switching permanent magnet machine with 12 stator poles and 14 rotor poles. Firstly, the machine design parameters are studied addressing on high output torque and its flux distribution is also investigated by finite-element method (FEM) analysis. Then a simplified lumped parameter magnetic circuit model is built up for analyzing design parameters. And a design procedure is also presented. The analytically designed machine is verified by FEM simulations.

Index Terms — Finite element method, permanent magnet machine, magnetic circuits, flux-switching, high torque.

I. NOMENCLATURE

- $B_r$: Magnet remanence
- $B_t$: Average flux density in stator tooth top
- $c_s$: Ratio of stator tooth width to stator pole pitch
- $D_o$: Machine outer diameter
- $F_{pm}$: Magnet MMF
- $g$: Airgap length
- $H_{ra}$: Rotor iron-back thickness
- $H_{sa}$: Stator iron-back thickness
- $H_t$: Stator tooth height
- $H_r$: Rotor tooth height
- $J$: Current density
- $k_f$: Winding factor
- $k_{pm}$: Magnet temperature coefficient
- $k_f$: Flux leakage factor
- $L$: Machine active axial length
- $l_{pm}$: Magnet thickness
- $P_g$: Permeance in airgap
- $P_{gi}$: Magnet inner air-leakage permeance
- $P_{gi}$: Magnet outer air-leakage permeance
- $P_{pm}$: Magnet permeance
- $P_r$: Rotor pole number
- $P_{br}$: Rotor back-iron permeance
- $P_{tr}$: Rotor tooth permeance
- $P_s$: Stator pole number
- $P_{br}$: Stator back-iron permeance
- $P_{st}$: Stator tooth permeance
- $S$: Electrical loading
- $T$: Torque
- $W_{rt}$: Rotor tooth width
- $W_s$: Stator slot opening
- $W_{st}$: Stator tooth width
- $\lambda$: Ratio of stator inner diameter to outer diameter
- $\tau_{r}$: Rotor pole pitch
- $\tau_{s}$: Stator pole pitch
- $\mu_0$: Permeability of free space
- $\mu_{pm}$: Magnet relative permeability
- $\omega$: Machine synchronous angular speed

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II. INTRODUCTION

Flux-switching permanent magnet (FSPM) machines, having PMs in the stator with doubly salient stator and rotor structure like a switched reluctance machine combine the advantage of a conventional PM machine and a switched reluctance machine. They have therefore high reliability, high torque/ power density and relatively high efficiency, hence preferable for reliability premium applications. Today FSPM machines have been presented for different applications, such as in aerospace, automotive and wind energy applications [1]-[3]. Several papers have investigated different FSPM machines with various stator and rotor pole combinations and their characteristics [4]-[11]. In [4] and [9] a FSPM machine with 12 stator poles and 14 rotor poles (12/14 poles) as shown in Fig. 1 has been investigated. Compared with a 12/10 pole machine, this machine can provide higher torque density with less torque ripple.

Today FSPM machines are generally designed as an initial machine, in which $H_{sb} = l_{pm} = W_{rt} = W_s = \tau_s / 4$ as shown in Fig. 2, thereafter the optimal parameters and/or performance were studied by either finite element method (FEM) simulations or lumped parameter magnetic circuit model [8] [11][14]. Such initially designed FSPM machines usually have highly saturated stator iron teeth that is normally beneficial for a 12/10 pole machine to improve the output torque. But for a 12/14 pole machine, the high saturation will lead to a torque decrease due to the high flux leakage between the stator and rotor [9]. So a new approach is required to design a high-torque 12/14 pole machine. This paper introduces a simplified lumped parameter magnetic circuit model to analytically design the machine. Firstly the machine design parameters are studied addressing on high output torque. Then the flux distribution of a typical 12/14 FSPM machine is investigated by FEM simulations, based on which a lumped parameter magnetic circuit model is built up for finding optimal design parameters. Finally, the analytically designed machine is verified by FEM simulations.

Fig. 1. Cross section of a 12/14 pole machine.
Fig. 1 shows the machine construction. Each phase winding of the machine consists of four coils and each coil is concentrated around two stator teeth with a magnet inset in between. The magnets are circumferentially magnetized and the magnetization is reversed in polarity from one magnet to the next. For each phase the flux in coils 1 and 2 are respectively the same as that in the corresponding phase coils 3 and 4 due to the symmetrical machine construction. The coil-flux linkage of each phase (the summary of four coils) is essentially sinusoidal with respect to the rotor position and has a period of \( \tau \) as shown in Fig. 3. And it reaches the peak value when the rotor is at the \( d \)-axis position as shown Fig. 4 (a). At this position the fluxes in the four coils of the phase are the same, as can be seen in Fig. 5 in which the fluxes in coils A1, A2, A3 and A4 are the same.

### IV. MACHINE DESIGN

#### A. Design parameters

If neglecting machine losses the torque of a 12/14 pole FSPM machine can be expressed as [9]

\[
T = \frac{\sqrt{2} n^2}{4 P} \sigma B_t L D_v^2 S c \eta.
\]

where \( B_t \) is the average flux density in the stator tooth tops at the \( d \)-axis position, and \( \sigma \) is the leakage factor representing the effective flux for torque production at the \( d \)-axis position and evaluated here by

\[
k_{\sigma} = \frac{\Phi_{p3} - \Phi_{p4}}{\Phi_{p3}}
\]

where \( \Phi_{p3} \) and \( \Phi_{p4} \) are respective the flux through the teeth \( P3 \) and \( P4 \) in Fig. 8.

The parameters \( D_v \) and \( L \) are generally constrained by the available volume of a specific case and are therefore fixed. In this paper they are respectively 100 mm and 200 mm.

\( B_t \) is an important design parameter. Ideally without considering iron saturation, the higher \( B_t \), the higher torque from (1) would be produced. In reality, along with an increase of \( B_t \) the value of \( k_{\sigma} \) will decrease because of the increased iron saturation. This has been proven by FEM analysis as presented in Fig. 6, in which \( B_t \) is varied by using different magnet materials with various \( B_r \) from 0.6 - 1.2 T, whilst keeping the machine dimension parameters unchanged. Since the output torque depends on both \( B_t \) and \( k_{\sigma} \), their product value that directly indicates the torque capability of the machine is also shown in the figure. It is observed that the product reaches its peak value when \( B_t \) is 1.8 ~ 1.9 T. With a further increased \( B_t \) the leakage flux \( \Phi_{p4} \) increases more than the total flux \( \Phi_{p3} \) due to the iron saturation as shown in Fig. 7. As consequence, the effective flux, \( \Phi_{p3} - \Phi_{p4} \), for torque production decreases, hence the leakage factor \( k_{\sigma} \) determined by (2). In this paper \( B_t \) is chosen to be 1.8 T, which is typically the saturation flux density of iron materials.
\( \lambda \) and \( c_s \) are selected as design variables here, then the other machine parameters can be expressed in terms of them as follows:

The tooth width is calculated as
\[
W_s = \tau_s c_s ,
\]
where \( \tau_s \) is the stator pole pitch and calculated by
\[
\tau_s = \pi D_s \alpha / P_s .
\]

The magnet width is determined by
\[
W_{pm} = (1-\lambda) D_s / 2 .
\]

The stator iron-back thickness \( H_{ib} \) is chosen so that the maximum flux density in the stator back iron is the same as \( B_r \). By so the iron does not get saturated and the stator winding can have the maximum available area. FEM simulations show that \( H_{ib} \) should be around 70% of \( W_s \) to avoid the saturation. The result is the same as that for a 12/10 pole machine [11]. The rotor iron-back thickness \( H_{rb} \) is chosen to be the same as \( H_{ib} \).

The height of the rotor tooth \( H_r \) determines the rotor saliency. Generally the reluctance torque of the machine is negligible. However, the output torque can be slightly increased with higher \( H_r \). The research based on a 12/10 pole machine shows the maximum torque is obtained when the rotor tooth height is around twice the stator tooth width [11]. This conclusion is employed for this machine design because of the similar construction of the two machines and the negligible reluctance torque value.

The magnet thickness \( l_{pm} \) may be started with a small initial value, for example, 1 mm here, so that \( B_r \) is less than 1.8 T. Its final value will be found out later in designed machine section. Then the electrical loading \( S \) is determined by the available copper area as
\[
S = A_{cu} k_s / (\pi D_s \alpha) .
\]

where \( A_{cu} \) is the copper area in the stator and given by
\[
A_{cu} = \pi \left( D_s / 2 - H_{ib} \right)^2 - \pi \left( \lambda D_s / 2 \right)^2 - P_s H_s \left( 2W_s + l_{pm} \right) .
\]

The rotor tooth width \( W_{rt} \), unlike the stator parameters, can be freely chosen without influencing the other design parameters. Although \( W_{rt} \) does not directly appear in torque equation (1), it affects the airgap leakage factor, further the leakage factor \( k_r \) and the output torque. The principle for selecting \( W_{rt} \) is to make \( k_r \) value as high as possible. For a 12/14 machine with \( l_{pm} = W_{rt} = \tau_s / 4 \), the optimal \( W_{rt} \) is found to be \( \tau_s / 3 \) [14]. In the discussed case both \( l_{pm} \) and \( W_{rt} \) are varied with different \( \lambda \) and \( c_s \), the rotor tooth width is chosen so that the left edge of the rotor tooth at the \( d \)-axis aligns with the left edge of the stator tooth as shown in Fig. 8. By so the maximum overlapped area between the stator and rotor teeth is obtained for the chosen \( W_{st} \) and \( W_{rt} \) at the \( d \)-axis. Then the rotor teeth width is determined by
\[
W_{rt} = 2W_{st} + l_{pm} - \tau_s / 2
\]
where
\[
\tau_r = (\lambda D_r - 2g) \pi / P_r .
\]

It should be noted that \( W_{rt} \) determined by (9) may not give the maximum leakage factor for the designed machines. If necessary, the rotor tooth width may be optimized by FEM analysis afterwards.

### B. Flux distribution

So far all the machine parameters in (1) are known except \( k_r \). Since \( B_r \) and \( k_r \) are calculated at \( d \)-axis position, it is of interest to investigate the flux distribution at that position. Fig. 5 shows the flux distribution of a typical 12/14 pole machine at the \( d \)-axis (phase \( a \) here). The four coils of phase \( a \) have the same flux linkage, and the flux in each coil is mainly from the three magnets near the phase coil as shown in Fig. 8 (a). So it is sufficient only to analyze the flux paths of these three magnets for evaluating \( B_r \) and \( k_r \). The flux distributions in the airgap between the stator and rotor teeth are approximated in Fig. 8 (b). The flux distributions in the arigap are slightly varied depending on the specific values of \( W_{st}, W_{rt}, \) and \( l_{pm} \).

### C. Lumped parameter magnetic circuit model

Based on the flux paths in Fig. 8, a lumped parameter magnetic circuit model of one fourth the machine is built up at no load condition as shown in Fig. 9. Compared with the model in [8] and [14] where half the machine is modeled, this model is simplified. In the presented model the end effect is not considered because the machine axial length is relatively long compared with its diameter \( L/D_s=2 \).

The permanent magnets are simply modeled as a MMF by (11) and their permeances are calculated by (13).
\[
F_{pm} = I_{pm} B_s / (\mu_p \mu_0)
\]

where \( B_s \) is the magnet remanence at the ambient temperature \( T_0 \) and it is determined by

### Fig. 8. (a) Flux distribution at the \( d \)-axis position. (b) Approximation of the flux paths in a plain form.
dependent parameter, obtained from (15) can be changed. After several iterations, and the field intensity (16) can be used as an approximation of the iron magnetization. The length of the corresponding iron part, where \( \mu \) is the permeability of the iron part and determined by iteration curve) is required for calculating the magnetic reluctance of the iron. In reality, it is very difficult to find an expression that can exactly represent the magnetic curve \( [12] \). Fortunately, equation (15) can be used to approximate the magnetization curve \( \Phi \) of each iron part is updated based on previous calculation as the following:

\[
B_x = B_x \left( 1 + k_{pm} (T_x - T_0) \right).
\]

(12)

where \( B_x \) is the magnet remanence at room temperature \( T_0 \).

\[
P_{pm} = \mu_{pm} H_i W_{pm} / l_{pm}
\]

(13)

The permeances of the iron parts, \( P_{ed}, P_{er} \) and \( P_{sh} \), are determined by

\[
P = \mu_i A / l.
\]

(14)

where \( A \) and \( l \) are respectively the cross-sectional area and the length of the corresponding iron part, \( \mu_i \) is the relatively permeability of the iron part and determined by iteration form the \( B-H \) curve of the lamination material.

To calculate the airgap permeances \( P_{ed}, P_{gil} \) and \( P_{gol} \) the method presented in [8] is employed here.

D. Approximation of Magnetization curve

An expression for the relation between the flux density and the field intensity (\( B-H \) curve) is required for calculating the magnetic reluctance of the iron. In reality, it is very difficult to find an expression that can exactly represent the curve. Fortunately, equation (15) can be used to approximate the magnetization curve [12].

\[
B_x(H_i) = \mu_i(H_i + M_s (\coth \frac{H_i}{a} - \frac{a}{H_i}))
\]

(15)

where \( M_s \) is saturation magnetization, and \( \alpha \) is a material dependent parameter, \( B_x \) and \( H_i \) are respectively the flux density and field intensity in the corresponding iron part.

By varying the values of \( M_s \) and \( \alpha \), the shape of the curve obtained from (15) can be changed. After several iterations, the curve with \( M_s = 1.5 \text{ MA/m} \) and \( \alpha = 550 \) shown in Fig. 10 can be used as an approximation of the iron magnetization curve.

\[
E. Magnetic circuit equations

Nodal analysis is employed to solve the magnetic circuit. Each permanent magnet is represented by a flux source and a flux resistance in parallel as shown in Fig. 9, in which \( \Phi_{pm} \) is given by

\[
\Phi_{pm} = P_{pm} F_{pm}
\]

(16)

There are totally 17 nodes in the magnetic circuit and the equations between the relationship of the these magnetic variables are established as

\[
\begin{bmatrix}
\Phi_1(1) \\
\Phi_2(2) \\
\Phi_3(3) \\
\Phi_4(17)
\end{bmatrix}
= 
\begin{bmatrix}
P(1,1) & P(1,2) & \ldots & P(1,17) \\
P(2,1) & P(2,2) & \ldots & P(2,17) \\
P(3,1) & P(3,2) & \ldots & P(3,17) \\
P(17,1) & P(17,2) & \ldots & P(17,17)
\end{bmatrix}
\begin{bmatrix}
F(1) \\
F(2) \\
\vdots \\
F(17)
\end{bmatrix}
\]

(17)

where \( \Phi_{(1)} - \Phi_{(17)} \) respectively represent the flux flowing into the corresponding nodes from the flux sources, here \( \Phi_{(1)} = \Phi_{(4)} = \Phi_{(5)} = \Phi_{pm} \) and \( \Phi_{(2)} = \Phi_{(3)} = \Phi_{(6)} = - \Phi_{pm} \), the others are zero.

\( P(m,n) \), \( m \neq n \), is the negative permeance value between nodes \( m \) and \( n \).

\( P(m,m) \) is the sum of the permeance of those branches connected to node \( m \). \( P(1), F(17) \) are respectively the magnetic potential at nodes 1-17.

When solving (17), the initial permeance value of each iron part is set \( \mu_i = 4000 \). Afterwards, \( \mu_i \) for \( k^{th} \) iteration of each iron part is updated based on previous calculation as the follows:

1. Calculating the magnetic field intensity over the corresponding iron part by

\[
H_i^{k+1} = \frac{\Delta F_i^{k+1}}{l}.
\]

(18)

where \( \Delta F_i^{k+1} \) is the magnetic potential drop over the corresponding iron part, this value can be calculated from the magnetic potentials at the nodes.

2. Updating \( \mu_i \) according with the magnetization curve in Fig. 10 by

\[
\mu_i = \frac{(H_i^{k+1} + M_s (\coth \frac{H_i^{k+1}}{a} - \frac{a}{H_i^{k+1}}))}{H_i^{k+1}}.
\]

(19)

3. Repeating the procedure with the updated \( \mu_i \) value until \( B_i \) and \( H_i \) are satisfactory with (15). \( B_i \) is obtained from

\[
B_i = \frac{\Delta F_i^{k} P}{A}.
\]

(20)

It should be mentioned that the magnetic saturation over teeth T4 is underestimated since the flux from P7 is not considered. Fortunately, the saturation is negligible due to the large airgap reluctance because of the small or even no overlapped iron area between teeth P6/P7 and T4 as shown in Fig. 8.
To calculate the maximum output torque from (1) with certain $\lambda$ and $c_s$, the value of $k_\sigma$ should be known when $B_t$ is 1.8 T. This is achieved by gradually increasing $l_{pm}$ based on the given initial value, then recalculating $W_{rt}$ from (9) and further all the permeances in Fig. 9. Thereafter, solving the model to figure out $\Phi_{p3}$ and $\Phi_{p4}$ and further $k_\sigma$ and $B_t$. Repeating the process until $B_t = 1.8$ T. Now $l_{pm}$ and $k_\sigma$ are known and the output torque can be evaluated by (1).

Fig. 11 presents the design procedure.

**G. Designed machine**

Fig. 12 shows the leakage factor as function of $\lambda$ and $c_s$. For each $c_s$ the value of $k_\sigma$ increases along with an increase of $\lambda$, and for each $\lambda$ there is an optimal $c_s$ where $k_\sigma$ reaches its maximum value.

Fig. 13 presents the output torque as function of $\lambda$ and $c_s$. There is an optimal $\lambda$ and $c_s$ giving the maximum output torque. Fig. 14 and Fig. 15 respectively show the maximum output torque with respect to split ratio $\lambda$ and stator tooth factor $c_s$. It is found that the optimal $\lambda$ is around 0.5 and $c_s$ is around 0.25 for the discussed case here. Table I lists the parameters of the designed machine.

It should be noted that the magnet demagnetization and the maximum allowable temperature of the winding insulation should be considered when selecting the current density. This is out of the scope and therefore is not discussed in this paper.

![Fig. 11. Machine design process.](image)

![Fig. 12. Leakage factor $k_\sigma$ as function of $\lambda$ and $c_s$.](image)

![Fig. 13. Output torque as function of $\lambda$ and $c_s$.](image)

![Fig. 14. Maximum output torque at different split ratio $\lambda$.](image)

![Fig. 15. Maximum output torque at different stator tooth factor $c_s$.](image)

**Table I Machine parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_o$</td>
<td>100 mm</td>
</tr>
<tr>
<td>$L$</td>
<td>200 mm</td>
</tr>
<tr>
<td>$g$</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.5</td>
</tr>
<tr>
<td>$c_s$</td>
<td>0.25</td>
</tr>
<tr>
<td>$P_s$</td>
<td>12</td>
</tr>
<tr>
<td>$P_r$</td>
<td>14</td>
</tr>
<tr>
<td>$W_{in}$</td>
<td>3.4 mm</td>
</tr>
<tr>
<td>$W_{st}$</td>
<td>3.2 mm</td>
</tr>
<tr>
<td>$H_{st}$</td>
<td>2.2 mm</td>
</tr>
<tr>
<td>$H_{rt}$</td>
<td>23 mm</td>
</tr>
<tr>
<td>$H_b$</td>
<td>6.4 mm</td>
</tr>
<tr>
<td>$l_{pm}$</td>
<td>2.4 mm</td>
</tr>
<tr>
<td>$B_r$</td>
<td>1.16 T</td>
</tr>
<tr>
<td>$k_\sigma$</td>
<td>0.6</td>
</tr>
<tr>
<td>$J$</td>
<td>4 A/mm$^2$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>150º</td>
</tr>
<tr>
<td>$k_{sat}$</td>
<td>0.0004 K$^{-1}$</td>
</tr>
</tbody>
</table>
V. FEM SIMULATION

To verify the result, the machine with the parameters given in Table I is investigated by 2D-FEM simulations, in which the $B-H$ curve in Fig. 10 is employed for the iron material and $B_r$ determined by (12) is set to 1.09 T to take the temperature influence into account. Fig. 16 shows the flux distribution of the machine at the $d$-axis position with no load ($J = 0$), from which $B_r$ and $k_o$ are obtained. Fig. 17 presents the output torque from the simulation. And Table II lists the results from both the lumped parameter magnetic circuit model and the FEM simulations. The torque calculated from the circuit model is about 3.3% higher than that from the FEM simulations. They match each other satisfactorily.

![Fig. 16. Flux distribution of the designed machine at no load](image)

![Fig. 17. Output torque of the machine from FEM](image)

| Table II Comparison of $B_r$, $k_o$ and $T$ |
|---------------------------------|-----------------|-----------------|
|                                | Analytical      | FEM             |
| $B_r$ (T) (no load)            | 1.8             | 1.76            |
| $k_o$ (no load)                | 0.67            | 0.69            |
| $T$ (Nm)                       | 25.4            | 24.6 (average)  |

VI. CONCLUSION

This paper has introduced a simplified lumped parameter magnetic circuit model for analytically designing a high-torque 12/14 pole FSPM machine. And the design procedure of how to find out the optimal design parameters is also presented. The design machine has been verified by FEM simulations.

VII. REFERENCES


VIII. BIographies

Anyuan Chen received the B.Sc. degree in electrical engineering from Wuhan Institute of Technology, Wuhan, China in 1991, and then worked as a senior electrical engineer at several companies. In 2004 he received the M.Sc. in Electrical Power Engineering from the Royal Institute of Technology (KTH), Stockholm, Sweden. Now he is working toward the Ph.D. degree in Norwegian University of Science and Technology (NTNU), Trondheim, Norway.

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