CONSTANT POWER LOAD CHARACTERISTIC’S INFLUENCE ON THE LOW-FREQUENCY INTERACTION BETWEEN ADVANCED ELECTRICAL RAIL VEHICLE AND RAILWAY TRACTION POWER SUPPLY WITH ROTARY CONVERTERS

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Abstract – Modern electric rail vehicles cause low-frequency rotor oscillations in the rotary converters of the Norwegian railway power supply. It is believed that one important factor for this instability is the constant power load characteristic of the vehicle seen from the network side. Additionally the vehicle low-frequency dynamics influence. This paper attempts to explain the physics of both the vehicle and the rotary converter low-frequency oscillatory modes. Further it is described qualitatively how interaction between these modes may lead to power system instability.

1. Introduction

Application of power electronic converters in electrical rail vehicles has increased in recent years. Fast and flexible control improves the vehicle’s performance. As a result, a number of electrical compatibility issues have been experienced world-wide [1]. One such compatibility issue is low-frequency power oscillations leading to power system instability. Table 1 shows an overview over recently experienced low-frequency oscillations in railway power systems. Typical oscillation frequency $f_{osc}$ is 10-30% of the respective power system’s fundamental frequency $f_s$.

Table 1 – Experienced low-frequency oscillations

<table>
<thead>
<tr>
<th>Case</th>
<th>$f_{osc}$ [Hz]</th>
<th>$f_s$ [Hz]</th>
<th>$f_{osc}/f_s$</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotary conv., Norway</td>
<td>1.6</td>
<td>16 $\frac{2}{3}$</td>
<td>0.10</td>
<td>[2]</td>
</tr>
<tr>
<td>Thionville, France</td>
<td>5</td>
<td>50</td>
<td>0.10</td>
<td>[3]</td>
</tr>
<tr>
<td>Washington, USA</td>
<td>3</td>
<td>25</td>
<td>0.12</td>
<td>[4]</td>
</tr>
<tr>
<td>Siemens test, Germany</td>
<td>7</td>
<td>50</td>
<td>0.14</td>
<td>[5]</td>
</tr>
<tr>
<td>Zürich, Switzerland</td>
<td>5</td>
<td>16.7</td>
<td>0.30</td>
<td>[6]</td>
</tr>
</tbody>
</table>

The list includes cases, such as [6], where the power supply system may be represented by a simple Thévenin equivalent. This means an ideal sinusoidal voltage source behind network impedance mainly determined by the overhead contact line and return circuit impedance together with the substation short circuit impedance. The power system low-frequency dynamics are then mainly determined by the vehicle. In other cases, such as reported in [2], the power supply system contribute with a significant dynamical behaviour at low frequencies. For instance the synchronous-synchronous rotary converters used for frequency conversion from the public three-phase 50-Hz network to single-phase 16 $\frac{2}{3}$-Hz railway network in Norway have a poorly damped electromechanical eigenfrequency around 1.6 Hz.

It is reported [7] to be challenging to make an advanced electrical vehicle compatible with these rotary converters. The most difficult operating condition is considered to be the vehicle operating on the end of a long line fed from one such rotary converter only. This system is focused in this paper and is illustrated in Fig.1.

The rotary converter is excited to oscillations as a result of generator electrical torque variations, i.e. given by active power changes in the 16 $\frac{2}{3}$-Hz system. The vehicle, though, has a mechanical power reference given by the desired motor torque and speed. As long as line power limitation is not active, this sets the electrical power demand which is seen from the power supply as constant power load ($CPL$).

This paper addresses this constant power load characteristic in view of the low-frequency oscillations in the power system. Simplified expressions for the low-frequency oscillatory modes of both the vehicle and the rotary converter are analytically derived. Based on this a qualitative stability criterion for the rotary converter is established. Further on, the interaction between the vehicle and the rotary converter is studied in principle by use of vehicle input admittance relative to the converter output impedance. A comparison to a complete simulation model including both vehicle and rotary converter is made before the results are discussed and concluded.
2. Power system load characteristics

In power system analysis loads are commonly characterized by their voltage dependency here described by the algebraic function in eqn. (1) [8]. This characteristic is true at least for modest amplitudes of change in the voltage \( U \) from the initial voltage \( U_0 \). The voltage dependent load power \( P \) is given by the initial power consumption \( P_0 \) and the voltage ratio raised to the power of the characteristic exponent \( MP \).

\[
P = P_0 \left( \frac{U}{U_0} \right)^{MP} \quad (1)
\]

This exponent defines the three main classifications of power loads:

- \( MP = 0 \): Load power demand is independent of voltage amplitude and the load may be referred to as a constant power load (CPL).
- \( MP = 1 \): Load power demand is proportional to voltage amplitude and the load may be referred to as a constant current load (CCL).
- \( MP = 2 \): Load power demand is proportional to voltage amplitude squared and the load may be referred to as constant impedance or resistance load (CRL).

Linearization by use of Taylor’s first order approximation of the exponent and assuming \( U_0 = 1 \) pu gives the relation between change in voltage \( \Delta U \) and change in current \( \Delta I \) as in eqn. (2). A CPL increases its current when the voltage drops, a CCL keeps the current unchanged and a CRL results in decreased current.

\[
\Delta I \approx P_0 \cdot (MP - 1) \cdot \Delta U \quad (2)
\]

In a single-phase system, a CPL must be understood based on averaging over one fundamental frequency period.

3. Oscillatory mode of the electrical rail vehicle

Most new electrical rail vehicles today utilize a four-quadrant voltage-source converter (VSC) behind a transformer as the interface to the rest of the power system as illustrated in Fig.1. This line inverter’s task is to transform the energy source for the motor VSC and the three-phase loop. It depends on the initial motor power \( P_0 \), the ratio of \( 0.5 \) is requested.

The resulting direct current \( I_{dc} \) from the PWM converter is fed into the DC-link. It is observed from several electrical locomotives that that the DC-link main capacitor typically has a discharge time of half a fundamental period. For a 16 ⅔-Hz vehicle this means that capacitor at rated DC-link voltage should contain an amount of energy enough to feed the motor side with rated current for 30 ms. To further reduce the resulting second harmonic DC-link voltage ripple, a resonant tank tuned at approximately 33.4 Hz [10] is used. The filter capacitor is typically in the range of 50 % to 100 % of the main capacitor increasing the total discharge time to for example \( T_i \approx 45 \) ms.

Standard control-theory tuning methods such as the symmetrical optimum [11] may be applied on the active power control loop in Fig.2. This results in a typical PI-controller integration time \( T_{ic} = 60 \) ms when a damping ratio of 0.5 is requested. \( K_{pi} \) is the proportional gain.

The motor current \( I_{dcm} \) acts as a disturbance in the control loop. It depends on the initial motor power \( P_{Mpu} \), \( U_{dc} \) and how the motor power is controlled relative \( U_{dc} \) variations, i.e. \( MP \) in eqn. (1). For linear analysis the feedback from \( U_{dc} \) to \( I_{dcm} \) can be represented by eqn. (2).

The two time constants \( T_{ic} \) and \( T_c \) are in the same range. Together they form an oscillatory mode. By neglecting the dynamics of the equivalent block, i.e. \( I_{dcm} = I_{dcm} \), an explicit expression for the eigenvalues \( \lambda_{1,2} \) describing this mode may be derived, see eqn. (3).

\[
\lambda_{1,2} = -\frac{Kp_{vc} + P_{Mpu0} (MP - 1)}{2T_c} \pm \sqrt{\frac{Kp_{vc} + P_{Mpu0} (MP - 1)}{2T_c} \left(-\frac{Kp_{vc}}{T_{ic}T_c}\right)} \quad (3)
\]
According to this, CPL characteristic reduces the damping. CCL is neutral and a CRL has a damping impact. Higher load increases the motor damping impact. This impact may be ideally cancelled by feeding forward $I_{dc\text{m}}$ into the PI-controller, i.e. add $I_{dc\text{m}}$ to $I_{dc}$.

It is important, though, to understand that the motor characteristic does not change the fact that the vehicle has a CPL characteristic seen from the power system.

4. Oscillatory mode of rotary converter

The rotary converter consists of a three-phase synchronous motor ($M$) mounted on the same stiff shaft as a single-phase synchronous generator ($G$) as shown in Fig.1. The synchronous motor’s connection to the three-phase network may be treated as a single-machine infinite bus (SMIB) system. A pronounced characteristic of such a SMIB system is the low-frequency oscillations given by the speed of the rotating machine and the angle displacement relative to a stiff voltage reference [8]. This is analogue to a mass-spring system. The rotating mass is given by the normalized inertia constant $H_{MG}$. The synchronizing torque linearization coefficient $K_{sM}$ describes the change in electrical torque $\Delta \tau_{MG}$ as function of the power angle $\Delta \delta_{M}$ and represents the spring constant. Hence the characterization ‘electromechanical’ is used.

The converter’s electromechanical mode is poorly damped due to the motor’s lack of explicit damper windings [12]. The remaining damping is here represented by the damping constant $D_M$. The basics of the converter’s electrical torque balance in island operating mode as illustrated in Fig.1 is shown by the upper part of the block diagram in Fig.3. $\omega_{MG} = 2\pi f = 2\pi \times 50 \text{ Hz}$ is the motor-side electrical angular speed.

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$$\Delta \alpha_{MG} = 2\pi f = 2\pi \times 50 \text{ Hz}$$

The lower part of the block diagram in Fig.3 describes the feedback from the rotor speed oscillations to the generator electrical torque $\Delta \tau_{MG}$. The electrical load is modelled according to eqn. (1). It is assumed a relation between the speed variation $\Delta \omega_{pu}$ and the induced generator voltage $\Delta U_{SG}$ according to eqn. (4), an increase in speed increases the voltage when $k_u > 0$.

$$\Delta U_{SG}(s) = k_u \cdot \Delta \omega_{pu}(s) \tag{4}$$

For the second-order system in Fig.3, an explicit expression for the rotary converter’s electromechanical oscillation mode taking the load voltage characteristic into account is shown in eqn. (5).

$$\lambda_{1,2} = -\frac{D_M \cdot \omega_{MG} + P_{eGpu0} \cdot k_{MP} \cdot (MP - 1)}{4 H_{MG}} \pm \sqrt{\left( \frac{D_M \cdot \omega_{MG} + P_{eGpu0} \cdot k_{MP} \cdot (MP - 1)}{4 H_{MG}} \right)^2 - \frac{K_{LM} \cdot \omega_{MG}}{2 H_{MG}}} \tag{5}$$

The undamped oscillation frequency is given by the rotating mass and the synchronizing torque coefficient. A CPL characteristic of the single-phase load is shown to reduce damping of the mode while a CCL is neutral and a CRL will act damping.

To prevent positive feedback from the single-phase network, increasing in rotary converter speed should not lead to decrease of generator electrical torque. Speed and torque variation should be in phase. Such a criterion is specifically important for the eigenfrequency. It results in a stability criterion as in eqn. (6) which the rail vehicle as a load should comply with to ensure stable interaction.

$$\text{Re} \left( \frac{\Delta \tau_{MG}(s)}{\Delta \omega(s)} \right) \geq 0 \tag{6}$$

5. Impedance and admittance considerations

The interface between electrical components is current $I$ and voltage $U$. Hence, the dynamical interface attributes may be described by how $I$ change when $U$ change or vice versa. The ratios $\Delta I/\Delta U$ and $\Delta U/\Delta I$ are commonly referred to as dynamical admittance $Y$ and dynamical impedance $Z$, respectively.

Assuming eqn. (4) to be valid and $I$ to be in phase with $U$ gives $\Delta P_{eGpu} \approx \Delta \tau_{MG} \Delta \omega_{pu} \propto \Delta U/\Delta I$ for a linearized system. The stability criterion in eqn. (6) may then be expressed as a load dynamical admittance criterion as in eqn. (7). The value of $k_u$ is of minor interest here as long as it is positive.

$$\text{Re} \left( \frac{\Delta I(s)}{\Delta U(s)} \right) = \text{Re} \left( Y(s) \right) \geq 0 \tag{7}$$

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$$Y_{CL} \approx P_{0} \cdot (MP - 1) \tag{8}$$

A CPL will show a negative dynamical admittance and will hence principally violate the stability criterion in eqn. (7). For a CCL there is no change in current resulting in a zero valued admittance. A CRL will show a positive dynamical admittance. This corresponds to the general understanding of CPLs in DC systems as described in [13].
Assuming the vehicle line converter to be lossless results in active power balance on the AC and the DC side as \( U_{ac}I_{ac} = U_{dc}I_{dc} \). If the vehicle keeps its power consumption constant independently of disturbance frequency, this leads to the dynamical input admittance \( Y_{CPL} \) of a non-dynamical CPL as described by eqn. (9). This corresponds to eqn. (8) when \( MP = 1 \) and \( U_{ac} = 1 \).

\[
Y_{CPL} = \frac{\Delta I_{ac}}{\Delta U_{ac}} = -\frac{P_{dc0}}{U_{ac0}^2}
\]

The vehicle active power control loop, however, has limited bandwidth. Its dynamics may be represented by the closed-loop transfer function of the block diagram in Fig.2. The equivalent first-order transfer function representing the current control loop is still assumed to be 1. For simplicity \( MP = 1 \). Hence the vehicle frequency-dependent low-frequency input admittance \( Y_v \) is described by eqn. (10).

\[
Y_v(s) = -\frac{P_{dc0}}{U_{ac0}^2} \cdot \frac{K_{p_{dc}}T_{i_{dc}}s + K_{p_{dc}}}{T_{ic}^2s^2 + K_{p_{dc}}T_{i_{dc}}s + K_{p_{dc}}}
\]

Based on the block diagram in Fig.3, the transfer function from \( \Delta o_{ac} \) to \( \Delta t_{CPL} \) may be found. An expression for the output impedance \( Z_c \) for the rotary converter as in eqn. (11) is achieved by taking advantage of the stability criterion from eqn. (6) into (7). The rotary converter’s oscillatory mode represents an increase in the impedance around its eigenfrequency.

\[
Z_c(s) = \frac{k_s s}{2H_{MG}s^2 + D_M\omega_Ms + K_{LM}\omega_M}
\]

These transfer function representations may be used in considerations of the interaction between the rotary converter and the rail vehicle.

6. System stability considerations

Together, the vehicle and the rotary converter represent a closed electrical circuit and control loop as illustrated in Fig.4 a) and b), respectively.

![Fig.4 – Closed electrical circuit a) and control loop b).](image)

The vehicle represents the load admittance \( Y_l = Y_v \). The converter impedance \( Z_c \) together with the overhead contact line impedance \((R + sL)\) represents the source impedance \( Z_s = Z_c + (R + sL) \). For simplification reasons, the line impedance here also includes the inner impedance in the generator, the converter station transformer impedance and the rail vehicle transformer impedance. The total system open loop transfer function is \( Z_cY_v \). The system will be unstable if the Nyquist contour of this transfer function encircles the critical point (-1, 0) in the complex plane. This corresponds to a loop gain larger than unity when the phase shift is 180° at a certain frequency. In [14] it is shown that stability can be guaranteed for a DC system if the magnitude of \( Z_c \) is always less than the magnitude of \( Z_s*Y_l < 1 \).

The rotary converter eigenfrequency at 1.6 Hz is easily observable and leads to an increase in source impedance. The vehicle low-frequency mode is observable as a drop and hence decrease the voltage operating point. The rotary converter’s power on the end of a 60 km long line fed from only one MW rail vehicle which is assumed to operate on 0.5 pu power. Numbers are given in Appendix. The case comprises a 6.4 MW rail vehicle which is assumed to operate on 0.5 pu power on the end of a 60 km long line fed from only one rotary converter. Vehicle voltage is 0.75 pu.

The system as it appears here is stable, i.e. \(|Z_c| < 1/Y_v\). The arrows in the figure indicate the gain margins. However, the following changes may be applied:

- An increase in line length shifts \( |Z_c| \) upwards. Hence the margin between \( Z_c \) and \( 1/Y_v \) will decrease.
- Increased line length will increase the line voltage drop and hence decrease the voltage operating point. This will according to eqn. (10) increase \( Y_l \) and decrease the margin between \( Z_c \) and \( 1/Y_v \).
- Increased initial load power will according to eqn. (9) and (10) increase \( Y_l \) and decrease the margin between \( Z_c \) and \( 1/Y_v \).
- The vehicle oscillatory mode damping an/or frequency is reduced. This will reduce the margin between \( Z_c \) and \( 1/Y_v \).
Additionally, it is experienced (but not described by the simplified models included in this paper) that increased line length may decrease both the frequency and damping of the vehicle oscillation mode. This may reduce the stability margins even more.

The difference between $|1/Y_{CPL}|$ and $|1/Y_v|$ shows the impact of the vehicle dynamics introduced in eqn. (10). At the vehicle oscillation frequency, its input admittance has a larger value. Amplification is also visible at the rotary converter frequency. This means that when the rotary converter oscillates, the vehicle will not only compensate the voltage oscillation by changing the current in its effort to keep the power constant. It will over-compensate by changing the current too much. The vehicle power will not be constant anymore, but oscillate opposite to the voltage and the converter rotor speed. This violates the stability criterions in eqn. (7) and (6) even more.

The system in this case is stable. If the margins, though, are sufficiently reduced the system will in the end become unstable. Without the rotary converter, there will not be an impedance peak at 1.6 Hz. This will increase the stability margins. Further increase of line length or power demand may result in instability at the vehicle oscillatory mode frequency or a voltage collapse (non-oscillatory system breakdown).

7. Simulation of complete model

A time simulation with a complete vehicle and rotary converter model is performed under the same operating conditions as was used in the study in section 6.

The complete simulation model is described more in detail in [15]. The vehicle includes synchronization controller (PLL), current controller, voltage and current measurement filters, full DC-link model and simplified model of the motor side. The rotary converter motor and generator are both modelled by fifth-order synchronous machine models in addition to their respective excitation systems and automatic voltage regulators.

Fig.6 shows simulation results for the complete vehicle and rotary converter model. All variables show oscillations at 1.6 Hz with increasing amplitude. The system is unstable. The different variables are plotted as deviation to their respective steady-state values if the system had been stable.

It is important to observe that the vehicle voltage and current oscillate opposite to each other. This is expected from the CPL behaviour. As can be seen, the vehicle power $\Delta P$ is not kept constant, but oscillates opposite to the line voltage $\Delta U$. Hence the DC-link voltage oscillates as well.

8. Discussion

The analytical considerations in this paper are based on several simplifications of the components and the system. Both vehicle and rotary converter are modelled as second-order systems. Several controllers in both the vehicle and the rotary converter are neglected. Additionally the AC side is treated as a DC system. Hence a deviation from both detailed simulation models and reality must be expected. Detailed simulation showed that the complete system was unstable while the simplified considerations did not.

The simplified impedance and admittance considerations here cannot be used for detailed stability investigations. A frequency domain method taking AC system characteristic of voltage and current phase angles into consideration as well is proposed in [6]. Reactive power and phase/frequency modulation cannot be neglected and it may have significant influence on the system, both by increasing and decreasing stability margins. For example, the reactive power shown in Fig.6 oscillates opposite to the active power and line voltage. This has a stabilizing impact as it reduces the line voltage amplitude oscillations. Reactive power oscillating against the line voltage in phase will have the opposite effect.

On the other side, the simplified models presented in this paper make it simpler to physically understand some phenomena in the vehicle-rotary converter interaction while others are neglected.

Similar low-frequency interaction between the source’s and the load’s dynamics is reported in [16] for uninterrupted power supplies (UPS) and active power factor correction (PFC) power supplies used in for example computers.
9. Conclusion

In this paper simplified second-order models are developed describing the basic low-frequency dynamics of an advanced electrical rail vehicle and a rotary frequency converter for railway power supply.

It is analytically shown how a constant power load may reduce damping of the respective components’ oscillatory mode’s frequency. A qualitatively stability criterion in view of the rotary converter’s poorly damped electromechanical eigenfrequency is proposed.

The rail vehicle acts as a constant power load seen from the power supply. Simplified impedance and admittance consideration indicates this characteristic as one reason for instability when a rail vehicle is supplied from the rotary converter. The stability margins at the rotary converter eigenfrequency are further reduced due to the vicinity in frequency of the vehicle’s own oscillatory mode.

The simplified considerations are assumed to give increased understanding of parts of the experienced interaction phenomena. The analytical results, however, differ in detail from a simulation of a complete system model. Simplified, the AC system is treated as a DC system. Influence from voltage and current phase variations is not taken into consideration. Additionally, several dynamical components in the simplified models are neglected. Hence deviations from detailed simulation models must be expected.

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References


Appendix

Vehicle: \(K_{\text{base}} = 1.67 \text{ pu}, T_{\text{base}} = 0.06 \text{ s}, T_{c} = 0.044 \text{ s}\). Rated power and base power 6.4 MW and DC-link voltage 2800 V. Main transformer impedance (0.044+j0.290) pu given a base power. \(MP = 1\).

Overhead contact line: (0.19+j0.21) Ohm/km

Rotary converter: Rated power and base power 4 MVA. Rated voltage and base voltage 4 kV. \(D_M = 0.015, H_{\text{base}} = 2*1.87 \text{ MWs/MVA}, K_{\text{base}} = 2.27 \text{ pu}\). Transient impedance inclusive transformer (0.032+j0.144) pu given base power and voltage. \(k_o = 1\).