State Space Averaging Modeling and Analysis of Disturbance Injection Method of MPPT for Small Wind Turbine Generating Systems

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Abstract—Based on a configuration with low cost and high reliability, disturbance injection method is employed to achieve the maximum power point tracking (MPPT) for the small wind turbine generating systems (SWTGS) in this paper. State space averaging method is used to model the whole system, and its nonlinear and linearization model are given. The choosing principle of two crucial parameters of disturbance magnitude \(d_m\) and angular frequency \(\omega\) are proposed by the frequency response analysis of the system. Experiment results show that the modeling of SWTGS and theoretical analysis of disturbance injection method of MPPT are correct and practical. Results obtained in the paper lay the foundation for the application of disturbance injection method of MPPT to SWTGS, and have theoretical significance and practical value in engineering.

Keywords-disturbance injection method; maximum power point tracking (MPPT); small wind turbine generating systems (SWTGS); state space averaging modeling

I. INTRODUCTION

The small wind power is one of the promising research and development fields in using the wind energy. The small wind turbine generating systems (SWTGS) of 1-10 kW, which can operate alone or connect with the power grid, can be installed in some areas of hills, grasslands, islands and even cities due to its flexibility. Though there are some 160 thousand sets of small wind power systems operating at present in China, its potential market is huge throughout the world[1].

Because of its small capacity and geographic location, it is important and essential for SWTGS to have low cost, high reliability as well as high efficiency. To meet these requirements, various system configurations and maximum power point tracking (MPPT) strategies have been reported [2]-[11]. The scheme proposed in [4] has the characteristics of simple structure and inexpensive hardware, for no position or speed sensors and complicated digital controllers are employed to realize MPPT control strategy or capture as much energy as possible from wind, and some analysis for the system performance has been done by means of both transfer function method and experiment. In this paper, state space averaging method is used to model the proposed SWTGS and its theoretical analysis of disturbance injection method of MPPT is carried out. As a result, the choosing principle of two crucial parameters of disturbance magnitude \(d_m\) and angular frequency \(\omega\) are given. Finally experiment verifies the correctness of system modeling and theoretical analysis for SWTGS.

II. DISTURBANCE INJECTION METHOD OF MPPT[4]

The configuration of SWTGS shown in Fig. 1 has the advantage of low cost and high reliability because it consists of permanent magnet synchronous generator, diode rectifier and chopper, and no speed sensors are needed to track the maximum output power of wind turbine. Output power or current of chopper is adjusted by controlling its duty due to almost constant voltage across the battery.

A disturbance injection method of MPPT for SWTGS is shown in Fig. 2 and 3. A sine disturbance signal with certain magnitude and frequency is injected to the chopper, and output current is sampled at the time of \(\pi/2\) and \(3\pi/2\) cycle, respectively \(I_1\) and \(I_2\). The control law can be described by the following equations

\[
\begin{align*}
d & = d_o + d_m \sin \omega t \\
d_o & = K \int (I_1 - I_2) dt
\end{align*}
\]

Suppose that the phase lag between output current and injected disturbance signal is \(\phi\). Because of the sampling time of output current, \(\phi = (2n+1)\pi/2\) when the system gets the maximum output power point, where \(n\) is integral. The
precondition for (1) is that: $\varphi \in (-\pi/2 + 2n\pi, \pi/2 + 2n\pi)$ if the system is operating at area A; $\varphi \in (-\pi/2 + 2n\pi, 3\pi/2 + 2n\pi)$ if operating at area B. When this condition is not satisfied, $d_0$ in (1) will change into

$$d_0 = -K \int (I_i - I_2) dt$$

(2)

Disturbance magnitude $d_0$ and angular frequency $\omega$ are two critical parameters of the disturbance injection method of MPPT for SWTGS. If $d_0$ is too big, large current fluctuation will occur in the system; If $d_0$ is too small, it will be not easy to detect the current difference at sample times, and not beneficial to get better control performance. If $\omega$ is too big, the phase lag of system will increase, and system realization will become difficult; If $\omega$ is too small, the dynamic response of the system is slow and control performance gets bad. The choosing of $d_0$ and $\omega$ will be discussed in the following sections.

III. STATE SPACE AVERAGING MODELING OF SWTGS

A. Electrical Circuit Model

For simplicity, the following assumptions are made for the system: 1) the property of wind turbine is simulated by a DC motor; 2) the property of synchronous generator and rectifying bridge with diodes is approached by a DC generator; 3) Boost chopper is employed; 4) the battery property is equivalent to an ideal voltage source in series with a resistor; 5) the property of inverter and load is similar to a resistor. Based on the assumptions above, the equivalent circuit of SWTGS is shown in Fig. 4.

B. Mathematic Model

Since it is difficult for the conventional method to be used to model the electrical circuit with switching devices, state space averaging method is employed here to model it [12]. Two main steps of state space averaging method are: 1) the individual state equations which correspond to ‘on’ or ‘off’ state of the system are obtained respectively; 2) the total state equation is acquired by weighing these individual state equations.

Assume that switching device and diodes have the same voltage drop $V_d$ when they are ‘on’ state, and the current of inductor of chopper is continuous (duty is $d$). If current of inductors, voltage of capacitors and angular speed of generator are chosen as state variables, the state equation of the whole system is obtained by state space averaging method

$$X = \begin{bmatrix} \frac{R_L}{K_m} & -\frac{K_m}{R_L} & 0 & 0 & 0 & 0 \\ 0 & \frac{R_L}{K_m} & -\frac{K_m}{R_L} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{C_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{C_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{C_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{R_C C_1} \end{bmatrix} U + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} U$$

(3)

$$i_d = \begin{bmatrix} i_u \\ \alpha_n \\ i_c \\ u_i \\ i_k \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} U$$

When the system gets into steady state, its output current is

$$i_d(\infty) = \frac{(1-d) \frac{R_L}{K_m} E - (1-d)^2 \frac{R_i}{R_s + R_L} U - (1-d) V_d}{R_s + \left(\frac{K_m}{K_s}\right)^2 R_a + (1-d)^2 \frac{R_i R_L}{R_s + R_L} + R_i}$$

(4)

C. Linearization Model

The system described by (3) is a nonlinear one, for it contains the product of variables. Equation (3) is linearized near its equilibrium point for dynamic analysis. Suppose that the system is at equilibrium state, we have $d=d_0$, $x=x_0$, $i_d=i_{d0}$ $A(d)=A(d_0)$. If a small disturbance $\Delta d$ near equilibrium point is injected into the system, then $d=d_0 + \Delta d$, $x=x_0 + \Delta x$, $i_d=i_{d0} + \Delta i_d$, $A(d)=A(d_0) + A(\Delta d)$. Substitute them into (3), and use $\Delta d$ and...
\[ \Delta i_d = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} \Delta X \]

\[ \Delta X = \begin{bmatrix} -\frac{R_s}{L_m} - \frac{K_x}{J} & 0 & 0 & 0 \\ \frac{K_x}{J} & 0 & 0 & 0 \\ 0 & \frac{K_x}{L_s} & -\frac{1}{L_s} & 0 \\ 0 & 0 & \frac{1}{C_1} & 0 \end{bmatrix} \Delta X + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta d \]

\[ \Delta i_d = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{R_s} & \frac{1}{R_s} \end{bmatrix} \Delta X \] (5)

its transfer function is

\[ \Delta d(s) = \frac{b_0 s^5 + b_1 s^4 + b_2 s^3 + b_3 s^2 + b_4 s + b_5}{a_0 s^5 + a_1 s^4 + a_2 s^3 + a_3 s^2 + a_4 s + a_5} \] (6)

where, coefficients are listed in Appendix 1.

IV. CHARACTERISTICS ANALYSIS

A. Steady State Analysis

Since equation (4) is a second order function of \( d \), its maximum output current \( i_d^* \) exists and is unique. Neglect of \( (1-d)^2 \) term in denominator, optimal \( d^* \) and maximum output current \( i_d^* \) are obtained respectively

\[ d^* = 1 - \frac{K_m E - V_d}{2 \left( \frac{R_s}{R_s + R_d} \right) U_d} = 1 - \frac{K_m E}{2K_m U_d} \] (7)

\[ i_d^* = \frac{\left( \frac{K_m E - V_d}{4U_d} \right)^2}{\left[ R_s + \left( \frac{K_m}{K_n} \right)^2 (R_s + R_d) \right] R_x + \frac{\left( \frac{K_m E - V_d}{4U_d} \right)^2}{\frac{K_m}{K_n} \left( R_s + R_d \right)}} \] (8)

It can be seen from (7) that \( d^* \) mainly depends on four parameters of \( E, U_d, K_n \), and \( K_m \). If \( E \) increases or decreases, \( d^* \) will decrease or increase, i.e., \( d^* \) varies with the wind speed. Equation (8) shows that \( i_d^* \) has something to do with not only the four parameters mentioned above, but \( R_{m}, R_{d} \) and \( R_{j} \) as well. In a word, equation (7) and (8) can be conveniently used for the system analysis and design.

B. Frequency Response Analysis

When parameters of the experimental system are used and \( E=100V \), bode diagram of linearization model is shown in Fig. 5. Similar bode diagrams can be obtained for different \( E \).

It can be seen from Fig. 5 that: 1) when \( d_0 \) changes from 0.6 to 0.7, notable change happens in bode diagram, i.e., \( d_0^* \) is in between 0.6 and 0.7; 2) when \( \varphi=90^\circ \), the system gets into equilibrium state. Equilibrium point is usually different from optimal point, and the difference between them relies on \( \varphi \); 3) When \( \omega \) is relatively small, if initial \( d_0<0.6 \), \( d_0 \) will increase and approach to \( d_0^* \) under the direction of controller, because \( 0^\circ \leq \varphi <90^\circ \); if initial \( d_0>0.7 \), \( d_0 \) will gradually approach to \( d_0^* \), because \( 90^\circ <\varphi \leq 180^\circ \); 4) when \( \omega \) is relatively large, \( d_0 \) will approach to certain equilibrium point. Because bode diagrams are very near, the equilibrium point is usually far from optimal point; 5) magnitude response is relatively smooth in lower frequency.

In summary, considering together the dynamic performance of the system, the choosing principle of disturbance magnitude \( d_m \) and angular frequency \( \omega \) is: 1) \( \omega \) is as large as possible when it is guaranteed that the system will acquire or approach to \( d_0^* \); 2) \( d_m \) depends on magnitude response and allowed fluctuation of system.

C. Phase Lag Analysis

If there exists phase lag in the system, its phase response will move down. If angular frequency \( \omega \) is given, \( d_0 \) will go away from \( d_0^* \), and output current will drop. Phase lag in the system comes from system itself and current detection circuit. Low pass filter (LPF) is introduced in current detection circuit to suppress current fluctuation. When angular frequency \( \omega \) is given, phase lag of LPF can be calculated. Phase lag compensation of LPF can be carried out by changing the sampling time of output current.

V. EXPERIMENTAL RESULTS

In experimental system, DC motor with additional exciting current is used to simulate the wind turbine; permanent magnet synchronous generator from wind mill is employed; switching device is MOSFET and its switching frequency is 40 kHz; DSP acts as controller; triangle waveform generating circuit is used...
to modulate pulse width; battery group consists of 4 single batteries of 12V. Parameters of the experimental system are listed in Table 1.

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<td>Rd (Ω)</td>
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A. Steady State Characteristics

When E is given, and d changes from 0.1 to 0.8 with the interval of 0.1, output current for each d forms a curve of steady state characteristics for system. Different curves for different E as well as those for analytic model are shown in Fig. 6. It reaches a conclusion from Fig. 6 that analytic model matches the experiment result apart from small d. In fact, the chopper usually works at the duty d of above 0.3.

B. Maximum Power Point Tracking

According to the choosing principle of disturbance parameters, ω=10×2π rad, d_m=0.05 and K=10. Steady state d and i_d are shown in Fig. 7, where E=160V. It shows that the closed loop control system achieves the maximum power control, where d^0 and i_d^0 are 0.60 and 6.4A respectively. When E has a sudden change from 160V to 80V, d^0 gradually converges to a new optimal duty of 0.75, where new optimal output current is 1.7A. Waveforms of variables are shown in Fig. 8. This shows that the controller is able to search optimal point ‘on line’ and make the system generate the maximum power all the time even when wind speed changes.

C. Performance Improvement

When compensation phase matches the phase lag from current detection device, the maximum output current can be obtained. The optimal compensation phase is about 30º, the maximum output current is shown in Fig. 9. In fact, when E=160V, more current of 0.5A (power 24W) is obtained, i.e., the efficiency of electricity generation is improved by approximately 7.7%.

VI. CONCLUSION

SWTGS has found certain applications and has a large potential development all over the world. Because of the geographic locations and working conditions, its property of

Fig. 6. Steady state relationship between output current and duty

Fig. 7. Waveforms of MPPT (E=160V)

Fig. 8. Waveforms of MPPT (E=160V drop to 80V)

Fig. 9. Performance improvement of phase lag compensation
Acknowledgment

The authors would like to acknowledge professor Ishida and Yamamura at MIE University, Japan for their helpful advice.

References


Appendix 1

\[
\begin{align*}
&b_1 = K_1 C_1 L_1 C_1 d_0 I_{d0} C_1 \\
&b_2 = K_1 (C_1 K_2 C_1 d_0 I_{d0} C_1 + L_1 K_3 d_0 C_1 + C_1 K_4 R_1 C_1 C_1 + C_1 K_5 R_1 C_1 C_1 + C_1 K_6 R_1 C_1 d_0 + C_1 K_7 R_1 C_1 C_1 + C_1 K_8 R_1 C_1 d_0) \\
&b_3 = K_1 (C_1 K_9 R_1 C_1 C_1 + C_1 K_10 R_1 C_1 C_1 + C_1 K_11 R_1 C_1 d_0 + C_1 K_12 R_1 C_1 C_1 + C_1 K_13 R_1 C_1 d_0) \\
&b_4 = K_1 (C_1 K_14 R_1 C_1 C_1 + C_1 K_15 R_1 C_1 C_1 + C_1 K_16 R_1 C_1 d_0 + C_1 K_17 R_1 C_1 C_1 + C_1 K_18 R_1 C_1 d_0) \\
&b_5 = K_1 (C_1 K_19 R_1 C_1 C_1 + C_1 K_20 R_1 C_1 C_1 + C_1 K_21 R_1 C_1 d_0 + C_1 K_22 R_1 C_1 C_1 + C_1 K_23 R_1 C_1 d_0) \\
&b_6 = K_1 (C_1 K_24 R_1 C_1 C_1 + C_1 K_25 R_1 C_1 C_1 + C_1 K_26 R_1 C_1 d_0 + C_1 K_27 R_1 C_1 C_1 + C_1 K_28 R_1 C_1 d_0)
\end{align*}
\]