AN ADAPTIVE DIRECT POWER CONTROL FOR MULTI-TERMINAL HVDC TRANSMISSION SYSTEM UNDER UNBALANCED CONDITIONS

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ABSTRACT
In this paper, the model-based Direct Power Controller (DPC) [1] is proposed for a Multi-terminal High Voltage Direct Current (HVDC) using Voltage Source Converter (VSC), which is able to control directly the instantaneous power and reactive power. Moreover, this DPC is able to handle the unbalanced network condition, that is unequal AC voltage due to unbalanced power supplies, unbalanced loads and unavoidable faults in the AC side. In addition the DPC includes an adaptation mechanism to cope with uncertain parameters. Numerical simulation has been performed to illustrate the merits of the proposed solution.

Index Terms— DPC, HVDC, Multi-terminal

1. INTRODUCTION
Despite Line-Commutated Converters (LCC), that is Current Source Converters (CSC), have been widely using for High Voltage Direct Current (HVDC) transmission and the control schemes are well studied, the Voltage Source Converter (VSC), using controllable switches like IGBTs, are attracted more attention due benefits features such as: (i) the active and reactive power exchange can be controlled flexibly and independently, which a sinusoidal currents at unity power factor are produced; (ii) no commutation failure problem; (iii) no communication required between two stations (iv) high-quality DC voltage, which a smaller capacitor is required [2]. However all this benefits are not necessary achieved under unbalanced network condition, a quite common condition in real operation [1].

Over a past few years, there have been many studies on the control of the Pulse-Width Modulation (PWM) converter under unbalanced conditions. In [3] is proposed to control the positive and negative sequence currents in a positive Synchronous Reference Frame (SRF), while in the [4] are used two SRF, however the process of extracting positive and negative-sequence components (both current and voltage) involves considerable time delay, this problem is minimized with the strategy proposed in [5], where a main and auxiliary controllers are proposed without any sequence separation. In [6] is proposed a dual current regulator with oscillating reference signals in a hybrid SRF in which also a current reference calculation is proposed. All the above controller can be identified as those controlling the current. But are also those who control directly the power like Direct Power Control (DPC), this algorithm is a fairly and simple control strategy which guarantee direct and effective regulation both active and reactive power, however the most of DPC algorithms assumed that source voltage is balanced [1].

On the other hand, VSC are well suited for operation in parallel, because it has proposed for the application in multi-terminal HVDC(M-HVDC) for transmission systems. In this paper, the model-based DPC [1] is proposed for a M-HVDC using VSC, which is able to control directly the instantaneous power and reactive power. This DPC proposed is able to handle the unbalanced network condition. In addition the DPC includes an adaptation mechanism to cope with uncertain parameters. Numerical simulation has been performed to illustrate the merits of the proposed solution.

2. SYSTEM MODEL
A Multi-terminal VSC-HVDC is an array which basically consists of three or more VSC converters, an schematic system model is depicted in Fig.1, directly connected by means of a common DC link. Each converter is coupled with AC network, via inductors $L$ while a capacitor $C$ is connected in the DC-side. For this work, without loss of generality since results can be extended for any number of rectifiers and any number of inverters, the VSC$_1$ is used as rectifier while the VSC$_2$ and VSC$_3$ are used as inverter. The average model of the system VSC$_1$ in $\alpha\beta$ coordinates, depicted in Fig.1, is described by:

$$\frac{d x_{\alpha\beta}}{dt} = -\theta_{\alpha\beta} + v_{\alpha\beta}$$

$C \frac{d}{dt} \left( \frac{v_{C1}^2}{2} \right) = \theta_{\alpha\beta}^T i_{\alpha\beta} - v_{C1} (i_{L1} + i_{L2})$

$\theta_{\alpha\beta} = \frac{v_{C1} i_{\alpha\beta}}{2}$
while the average model of the system VSC2 in αβ coordinates, is described by:

\[
L \frac{d i_{\alpha\beta 2}}{dt} = -\theta_{\alpha\beta 2} + v_{S\alpha\beta 2} \tag{3}
\]

\[
C \frac{d}{dt} \left( \frac{v_{\alpha\beta 2}^2}{2} \right) = \theta^T_{\alpha\beta 2} i_{\alpha\beta 2} + v_{C\alpha} (i_{\alpha 1}) \tag{4}
\]

and the average model of the system VSC3 in αβ coordinates, is described by:

\[
L \frac{d i_{\alpha\beta 3}}{dt} = -\theta_{\alpha\beta 3} + v_{S\alpha\beta 3} \tag{5}
\]

\[
C \frac{d}{dt} \left( \frac{v_{\alpha\beta 3}^2}{2} \right) = \theta^T_{\alpha\beta 3} i_{\alpha\beta 3} + v_{C\beta} (i_{\beta 2}) \tag{6}
\]

where \( i_{\alpha\beta 1} = [i_{S\alpha 1}, i_{S\beta 1}]^T \), \( i_{\alpha\beta 2} = [i_{S\alpha 2}, i_{S\beta 2}]^T \) and \( i_{\alpha\beta 3} = [i_{S\alpha 3}, i_{S\beta 3}]^T \) represent the line currents; \( v_{S\alpha 1} = [v_{S\alpha 1}, v_{S\beta 1}]^T \), \( v_{S\alpha 2} = [v_{S\alpha 2}, v_{S\beta 2}]^T \) and \( v_{S\alpha 3} = [v_{S\alpha 3}, v_{S\beta 3}]^T \) represent the source voltages and \( u_{\alpha\beta 1} = [u_{\alpha 1}, u_{\beta 1}]^T \), \( u_{\alpha\beta 2} = [u_{\alpha 2}, u_{\beta 2}]^T \) and \( u_{\alpha\beta 3} = [u_{\alpha 3}, u_{\beta 3}]^T \) are the duty ratios in the VSC1, VSC2, and VSC3, respectively; while \( v_{C\alpha} \), \( v_{C\beta} \) and \( v_{C\gamma} \) represent the capacitor voltages in rectifier and the inverters respectively. \( L \) and \( C \) are the inductance and capacitance. All along the document, bold type characters represent vector or matrices, while normal type scalars.

**Remark 2.1** This proposed approach is based on the averaged version of the system model. In this case its assumed that sufficiently high frequency is used to implement the switching control sequence \([\delta_1, \delta_2, \delta_3]^T \in [0, 1] \), so that the switching sector vector can be replaced by the duty ratios \([u_1, u_2, u_3]^T \in [-1, 1] \).

### 2.1. Transformation of the model in instantaneous power

The idea behind of DPC controllers consist in expressing the model in terms of the instantaneous reactive and active power \([1] \), instead of currents. Now, considering the following transformation \([7] \):

\[
\begin{bmatrix}
p_1 \\
q_1
\end{bmatrix} =
\begin{bmatrix}
v_{S\alpha 1} & v_{S\beta 1} \\
n_{S\alpha 1} & n_{S\beta 1}
\end{bmatrix}
\begin{bmatrix}
i_{S\alpha 1} \\
i_{S\beta 1}
\end{bmatrix} = R_1 i_{S\alpha\beta 1} \tag{7}
\]

Using the above transformation, the current dynamic, subsystem (1), can be expressed in terms of the active and reactive power that is:

\[
\begin{bmatrix}
L_1 \frac{d}{dt} p_1 \\
L_2 \frac{d}{dt} q_1
\end{bmatrix} =
\begin{bmatrix}
R_1 (-\theta_{\alpha\beta 1} + v_{S\alpha\beta 1}) - \sigma_1 J \\
0 1
\end{bmatrix}
\begin{bmatrix}
p_1 \\
q_1
\end{bmatrix} \tag{8}
\]

where \( \sigma_1 = \omega_0 L_1 \), \( \omega_0 \) is the fundamental frequency and \( J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \) the anti-symmetrical matrix.

### 2.2. Transformation of the model in instantaneous power in unbalanced case

The above representation (8), as most DPC approach, assumed that the voltage is balanced (noticed that \( v_{S\alpha\beta 1} = J \omega_0 v_{S\alpha\beta 1} \)). However, due the unavoidable conditions the unequal amplitude in AC voltage is
a common case in real operations, to consider this issue in the controller design it is necessary a more complete description, considering the next representation:

\[ v_{S\alpha f1} = v_{S\alpha f1,p} + v_{S\alpha f1,n} = e^{J_{lost}} v_{S1}^p + e^{J_{lost}} v_{S1}^n \]  \( (9) \)

where \( v_{S\alpha f1,p} \) and \( v_{S\alpha f1,n} \) represent the positive and negative sequence components of the source voltage while \( v_{S1}^p, v_{S1}^n \in \mathbb{R}^2 \) are the fundamental harmonic coefficients for the positive and negative representation of the source voltage, which is assumed unknown constants and \( e \) is a rotation matrix of the form:

\[ e^{J_{lost}} = \begin{bmatrix} \cos(\omega t) & -\sin(\omega t) \\ \sin(\omega t) & \cos(\omega t) \end{bmatrix} \]  \( (10) \)

Based on the above representation \( (9) \), the complete representation, which was proposed in [1], for unbalanced source voltage is:

\[ \begin{aligned} v_{S\alpha f1} &= J \omega_0 \phi_{\alpha f1} \\ \dot{\phi}_{\alpha f1} &= J \omega_0 v_{S\alpha f1} \end{aligned} \]  \( (11) \)

where \( \phi_{\alpha f1} = [\phi_{s1}, \phi_{f1}]^T = v_{S\alpha f1,p} - v_{S\alpha f1,n} \). Now, considering this more complete representation of \( v_{S\alpha f1} \), the subsystem (1) can be rewritten as:

\[ \begin{bmatrix} L \dot{p}_1 \\ L \dot{q}_1 \end{bmatrix} = R_1(-\dot{\theta}_{\alpha f1} + v_{S\alpha f1}) - \sigma_1 J \left[ \begin{array}{c} e_1 \\ \psi_1 \end{array} \right] \]  \( (12) \)

where \( e_1 \) and \( \psi_1 \) are defined as:

\[ \begin{bmatrix} e_1 \\ \psi_1 \end{bmatrix} = \begin{bmatrix} \phi_{s1} & \phi_{f1} \\ -\phi_{f1} & \phi_{s1} \end{bmatrix} \begin{bmatrix} i_{S1} \\ i_{Sf1} \end{bmatrix} \]  \( (13) \)

In a similar procedure, the subsystem (3) and (5) can be rewritten in order to express the dynamics in the active and reactive power term, that is \([p_2, q_2]^T\) and \([p_3, q_3]^T\) for the VSC2 and VSC3 respectively.

### 3. CONTROL DESIGN

The control objective for the rectifier VSC1 consists in design a controller to guarantee regulation of both active \( p_1 \) and reactive \( q_1 \) power toward a constant references \( p_1^* \) and \( q_1^* \) that is, \( p_1 \rightarrow p_1^*, q_1 \rightarrow q_1^* \) as \( t \rightarrow \infty \), in addition, for the normal operation of the VSC transmission line. Its proposed that the rectifier must maintained the capacitor voltage at a constant value \( v_{C1} \), that is, \( v_{C1} \rightarrow v_{C1,d} \). The control objectives for the inverter as well as objective of rectifier is guarantee regulation of active power, \( p_2 \) and \( p_3 \), and the reactive power \( q_2 \) and \( q_3 \).

For design control purpose the decoupling assumption is considered, that is its assumed that the power dynamics \( (12) \) responds much faster than the dynamics involving the capacitor voltage \( (2) \), this assumption allows to decoupled the design control in two loops, inner loop and outer loop.

#### 3.1. Inner loop for rectifier

It may proceed to apply the Energy Sharpening plus Damping Injection (ESDI), a procedure proposed in [8]. Following the ESDI procedure, a copy of the subsystem \( (12) \), where the state \([p_1, q_1]^T\) is replaced by the desired reference \([p_1^*, q_1^*]^T\), damping terms are included, as follows:

\[ \begin{bmatrix} L \dot{p}_1^* \\ L \dot{q}_1^* \end{bmatrix} = R_1(-\dot{\theta}_{\alpha f1} + v_{S\alpha f1}) - \sigma_1 J \left[ \begin{array}{c} e_1 \\ \psi_1 \end{array} \right] + K_1 \left[ \begin{array}{c} \tilde{p}_1 \\ \tilde{q}_1 \end{array} \right] \]  \( (14) \)

where \( \tilde{p}_1 \triangleq p_1 - p_1^* \) and \( \tilde{q}_1 \triangleq q_1 - q_1^* \) represents the active and reactive power errors, respectively. \( K_1 \) is a matrix which contain positive design parameters in the main diagonal which is used to introduce damping. Considering that \([p_1^*, q_1^*]^T = [0, 0]^T\) from \( (14) \) its obtained the control vector \( \theta_{\alpha f1} \), that is

\[ \theta_{\alpha f1} = v_{S\alpha f1} + R_1^{-1}( -\sigma_1 J \left[ \begin{array}{c} e_1 \\ \psi_1 \end{array} \right] + K_1 \left[ \begin{array}{c} \tilde{p}_1 \\ \tilde{q}_1 \end{array} \right] ) \]  \( (15) \)

Notice that \( \sigma_1 \) has been replaced by its estimate \( \tilde{\sigma}_1 \). Subsystem \( (12) \) in closed loop with the above controller \( (15) \) which yields the following error model

\[ \begin{bmatrix} L \dot{p}_1^* \\ L \dot{q}_1^* \end{bmatrix} = -K_1 \left[ \begin{array}{c} \tilde{p}_1 \\ \tilde{q}_1 \end{array} \right] + \sigma_1 J \left[ \begin{array}{c} e_1 \\ \psi_1 \end{array} \right] \]  \( (16) \)

where \( \tilde{\sigma}_1 \triangleq \sigma_1 - \sigma_1 \) represents the estimation error of \( \sigma_1 \).

#### 3.1.1. Adaptation law for \( \sigma_1 \)

The design of the adaptation law for \( \tilde{\sigma}_1 \) using a Lyapunov approach [9]. Following the quadratic function proposed in [1], that is

\[ W = L \tilde{p}_1^2 + L \tilde{q}_1^2 + \frac{1}{2\gamma} \tilde{\sigma}_1^2 \]  \( (17) \)

where \( \gamma \) is a positive design parameter representing the adaptive gain.

Its time derivative along the trajectories \( (16) \) is given by

\[ \dot{W} = -k_1 \tilde{p}_1^2 - k_1 \tilde{q}_1^2 + \tilde{\sigma}_1 \left[ \begin{array}{c} \tilde{p}_1 \\ \tilde{q}_1 \end{array} \right] J \left[ \begin{array}{c} \epsilon_1 \\ \psi_1 \end{array} \right] + \frac{1}{\gamma} \tilde{\sigma}_1 \tilde{\sigma}_1 \]  \( (18) \)

the time derivative\( (18) \) is made negative semidefinite by proposing

\[ \tilde{\sigma}_1 = -\gamma \left[ \begin{array}{c} \tilde{p}_1 \\ \tilde{q}_1 \end{array} \right] J \left[ \begin{array}{c} \epsilon_1 \\ \psi_1 \end{array} \right] = \gamma (\tilde{p}_1 \psi_1 - \tilde{q}_1 \epsilon_1) \]  \( (19) \)

Its possible to conclude that \( \tilde{p}_1 \rightarrow 0 \) and \( \tilde{q}_1 \rightarrow 0 \) asymptotically with LaSalles arguments as was shown in [1]. Notice that proposed controller requires the estimation of \( \phi_{\alpha f1} \), however is not accessible directly then is proposed to reconstructed in the same manner as [1] in the next section.
3.1.2. Estimation of $\phi_{a|b_1}$

The implementation of the controller (15) requires the vector $[\epsilon_1, \psi_1]^\top$, which implies the estimation of vector $\phi_{a|b_1}$. However this vector is not available directly, the proposed estimator consist in a copy of the system model (12) to which a damping term is added, that is,

$$\dot{v}_{S_{a|b_1}} = J\omega_0\phi_{a|b_1} + \lambda v_{S_{a|b_1}}$$

and

$$\dot{\phi}_{a|b_1} = J\omega_0 \dot{v}_{S_{a|b_1}}$$

where $\lambda$ is the estimation gain; $\dot{v}_{S_{a|b_1}}$ and $\dot{\phi}_{a|b_1}$ are the estimates of $v_{S_{a|b_1}}$ and $\phi_{a|b_1}$ respectively, and $\dot{v}_{S_{a|b_1}} = \ddot{v}_{S_{a|b_1}} - \dot{v}_{S_{a|b_1}}$.

The estimation error dynamics is given by

$$\dot{\tilde{v}}_{S_{a|b_1}} = J\omega_0\tilde{\phi}_{a|b_1} - \lambda \tilde{v}_{S_{a|b_1}}$$

(20)

where $\tilde{\phi}_{a|b_1} = \phi_{a|b_1} - \tilde{\phi}_{a|b_1}$ represents the estimation error of $\phi_{a|b_1}$.

Convergence of the estimator is again following a Lyapunov approach where the next quadratic function is used:

$$H = \frac{1}{2}\tilde{v}_{S_{a|b_1}}^2 + \frac{1}{2}\phi_{a|b_1}^2$$

Its time derivative along the trajectories of (20) yields the following semidefinite function

$$\dot{H} = -\lambda \tilde{v}_{S_{a|b_1}} \tilde{v}_{S_{a|b_1}}$$

based on the LaSalle arguments it can be concluded that $\tilde{v}_{S_{a|b_1}} \to 0$ and $\tilde{\phi}_{a|b_1} \to 0$ asymptotically as was shown in [1]. Notice that the estimator requires the knowledge of $\omega_0$.

4. INNER LOOP FOR INVERTERS

Following the same idea of ESDI procedure, its obtained the control vector $\theta_{a|b_2}$ and $\theta_{a|b_3}$ for the inverters

$$\theta_{a|b_2} = v_{S_{a|b_2}} + R_2^{-1}\left\{ -\hat{\sigma}_2 J \begin{bmatrix} \epsilon_2 \\ \psi_2 \end{bmatrix} + K_2 \begin{bmatrix} \ddot{p}_2 \\ \ddot{q}_2 \end{bmatrix} \right\}$$

(21)

and

$$\theta_{a|b_3} = v_{S_{a|b_3}} + R_3^{-1}\left\{ -\hat{\sigma}_3 J \begin{bmatrix} \epsilon_3 \\ \psi_3 \end{bmatrix} + K_3 \begin{bmatrix} \ddot{p}_3 \\ \ddot{q}_3 \end{bmatrix} \right\}$$

(22)

where $K_2$ and $K_3$ are matrix which contain positive design parameters in the main diagonal which is used to introduce damping. Notice that new definitions have been included ($R_2$ and $R_3$ according to (7), $\hat{\sigma}_2$ and $\hat{\sigma}_3$ according to (19) and $[\epsilon_2, \psi_2]^\top$ and $[\epsilon_3, \psi_3]^\top$ according to (13).

5. OUTER LOOP

This part of control deal with the voltage regulation objective. Its proposed that the rectifier regulate the capacitor voltage, and the same time computing the reference for the inner loop of the rectifier, that is $p_{1*}$. The voltage capacitor dynamic of subsystem (2), which is rewritten here, is describe by

$$C \frac{d}{dt} z_1 = \theta_{a|b_1}^T I_{S_{a|b_1}} - v_{C_1}(i_{L_1} + i_{L_2})$$

(23)

where $z_1 \approx \frac{v_{2*}}{2}$. The voltage capacitor dynamic of subsystem (4)

$$C \frac{d}{dt} z_2 = -\theta_{a|b_2}^T I_{S_{a|b_2}} + v_{C_2}(i_{L_1})$$

(24)

and the voltage capacitor dynamic of subsystem (6)

$$C \frac{d}{dt} z_3 = -\theta_{a|b_3}^T I_{S_{a|b_3}} + v_{C_3}(i_{L_2})$$

(25)

The dynamics (24) and (25) are substituted in dynamic (23), where $v_{C_1} = v_{C_2} = v_{C_3}$ is assumed, that is

$$3C \frac{d}{dt} z_1 = -\theta_{a|b_1}^T I_{S_{a|b_1}} - (\theta_{a|b_2}^T I_{S_{a|b_2}} + \theta_{a|b_3}^T I_{S_{a|b_3}})$$

(26)

Notice that the regulation objective can be restated as follows $z_1 \to v_{2*}^2$. As was mentioned before, it assumed that dynamics (1), (3) and (5) are much faster than (2), (4) and (6), therefore its assumed that active power and reactive power have reached to their reference, that is $p_1 = p_{1*}$, $p_2 = p_{2*}$, $p_3 = p_{3*}$ and $q_1 = q_{1*}$, $q_2 = q_{2*}$, $q_3 = q_{3*}$, and the parameter estimators have reached to their nominal values, that is $\hat{\sigma}_1 = \sigma_0$, $\hat{\sigma}_2 = \sigma_0 L$, $\hat{\sigma}_3 = \sigma_0 L$, and $\hat{\sigma}_1 = \sigma_0 L$ after a relative short time, based in the above assumptions the controller (15) can be simplified as

$$\tilde{\theta}_{a|b_1} = v_{S_{a|b_1}} - R_1^{-1} \omega_0 L \begin{bmatrix} \epsilon_1 \\ \psi_1 \end{bmatrix}$$

(27)

Similarly the (21)

$$\tilde{\theta}_{a|b_2} = v_{S_{a|b_2}} - R_2^{-1} \omega_0 L \begin{bmatrix} \epsilon_2 \\ \psi_2 \end{bmatrix}$$

(28)

and (22)

$$\tilde{\theta}_{a|b_3} = v_{S_{a|b_3}} - R_3^{-1} \omega_0 L \begin{bmatrix} \epsilon_3 \\ \psi_3 \end{bmatrix}$$

(29)

where the notation (\(
\tilde{\cdot}
\)) means that value (\(
\cdot
\)) have reached to their corresponding reference values. The controller (27) can be simplified as

$$\tilde{\theta}_{a|b_1} = v_{S_{a|b_1}} + \frac{\omega_0 L}{|v_{S_{a|b_1}}|^2} (v_{S_{a|b_1}}^T I_{S_{a|b_1}} \phi_{a|b_1} - v_{S_{a|b_1}}^T I_{S_{a|b_1}} \phi_{a|b_1})$$

(30)

where $I$ is the $2 \times 2$ identity matrix. The product $\theta_{a|b_1}^T I_{S_{a|b_1}}$ can be expressed as

$$\tilde{\theta}_{a|b_1} I_{S_{a|b_1}} = v_{S_{a|b_1}}^T I_{S_{a|b_1}} + \frac{\omega_0 L}{|v_{S_{a|b_1}}|^2} v_{S_{a|b_1}}^T I_{S_{a|b_1}} \phi_{a|b_1}^T I_{S_{a|b_1}}$$

(31)
where $v_{S_{a1}}^T J_{a1}$ and $i_{S_{a1}}$ can be rewritten as
\begin{equation}
v_{S_{a1}}^T J_{a1} = 2v_{S_{a1},p} J_{S_{a1},p}
\end{equation}
and
\begin{equation}
i_{S_{a1}} = \frac{1}{|v_{S_{a1}}|^2} (p_{1}^2 + q_{1}^2)
\end{equation}
 respectively. Notice that the products $\theta_{a2} i_{S_{a2}}$ and $\theta_{a3} i_{S_{a3}}$ can be rewritten in a similar form.

Direct Substitution of expressions $\theta_{a1} i_{S_{a1}}, \theta_{a2} i_{S_{a2}}$ and $\theta_{a3} i_{S_{a3}}$ in (26)
\begin{equation}
3C \frac{d}{dt} z_1 = p_1^* + f_1(p_1^*) + f_2(p_2^*) + f_3(p_3^*)
\end{equation}
where it using the fact that $p_1^* = v_{S_{a1}}^T i_{S_{a1}}, p_2^* = v_{S_{a2}}^T i_{S_{a2}}$ and $p_3^* = v_{S_{a3}}^T i_{S_{a3}}$
while
\begin{equation}
f_1(p_1^*) = \frac{2a_0}{|v_{S_{a1}}|} v_{S_{a1},p} (p_{1}^2 + q_{1}^2),
\end{equation}
\begin{equation}
f_2(p_2^*) = \frac{2a_0}{|v_{S_{a2}}|} v_{S_{a2},p} (p_{2}^2 + q_{2}^2),
\end{equation}
and
\begin{equation}
f_3(p_3^*) = \frac{2a_0}{|v_{S_{a3}}|} v_{S_{a3},p} (p_{3}^2 + q_{3}^2).
\end{equation}

The equation (34) can be rewritten in terms of $p_1^*$ if we consider the fact that the power balance between the rectifiers and inverters that is $p_1^* = -(p_2^* + p_3^*), \text{ and } f_1(p_1^*), f_2(p_2^*) \text{ and } f_3(p_3^*)$ are terms who contribute to the second harmonic component generated by the products $v_{S_{a1},p} J_{S_{a1},p}, v_{S_{a2},p} J_{S_{a2},p}$ and $v_{S_{a3},p} J_{S_{a3},p}$ notice that this second harmonics, which will appear in the DC voltage, grows with the power demand and is a consequence of the unbalance conditions, in the balance condition this terms must be zero.

In the (34) $z_1$ must be regulated via $p_1^*$ that acts as input control, the usual Proportional Integral (PI) controller can be warranty the regulation of $z_1$ of the following form
\begin{equation}
p_1^* = -k_p \frac{\text{d}}{\text{d}t} z_1 - k_i z_1
\end{equation}
where $z_1 = (z_1 - \frac{1}{\omega_c})$ and $k_p$ and $k_i$ are the proportional and integral gains, respectively and $\frac{1}{\omega_c}$ is the cutoff frequency of a low pass filter using to reduced the effect of distortion present in $z_1$

Notice that the regulation objective can only fulfilled in average, roughly speaking the controller will handle the DC component of the state, the fluctuation due the unbalance will be reduced using the enough bandwidth. The controller for the rectifiers is shown in the block diagram of the Fig. 2 and the block diagram for the inverter is similar but not include a PI controller to produce the reference of the active power $p_1$, these references for the inverters are fixed.

6. NUMERICAL RESULTS

The system has been implemented using PSCAD to test the performance of the controller. The parameter using are the following: $L=4.8$mH, $C=1100$mF the switching frequency was set in 2KHz. The source has a line voltage 20 kV at 50 Hz in the each AC side which is connected to the converters by means of delta-wye transformers.

The Fig.3 shows the active and reactive power for VSC$_1$, VSC$_2$, VSC$_3$ and the DC voltage respectively; the test was performed as followed: first, staring in the balanced operation for all converter and $p_{2*}=20$MVA and $p_{3*}=35$MVA are selected, the converter VSC$_2$ going from balance to unbalance due a single-line-ground fault at $t=0.3s$ while VSC$_1$ and VSC$_3$ going from balance to unbalance due 15% unbalance in one-phase input voltage a quite common condition in weak AC systems. At time $t=0.7$ a change in the reference $p_{2*}=40$MVA and $p_{3*}=50$MVA are selected, the reference for the reactive power ($q_{1*}=0, q_{2*}=0, q_{3*}=0$) is selected in zero during all the test. Notice both active and reactive power is regulated toward its references, and notice the DC voltage is maintained close to its reference ($v_d=50$KV) after some small transients due to the change of reference and unbalance cases, the introduction of the second harmonic in the DC voltage is observed after the unbalanced condition is presented as expected in dynamic description. The Fig. 4 shows the active and reactive power for VSC$_1$, VSC$_2$, VSC$_3$ and the DC voltage respectively first, staring in the balanced operation for all converter and $p_{2*}=20$MVA and $p_{3*}=35$MVA are selected again, the converter VSC$_1$ going from balance to unbalance due a single-line-ground fault at $t=0.3s$. At time $t=0.7$ a change in the reference $p_{2*}=40$MVA and $p_{3*}=50$MVA are
selected again, the reference for the reactive power ($q_1^* = 0, q_2^* = 0, q_3^* = 0$) is selected in zero during all the test, notice that the introduction of the second harmonic in the DC voltage is more evidently due more adverse condition however the controller can warranty the regulation both active and reactive power and the DC voltage.

7. CONCLUSIONS

This paper presented a DPC controller for a multi-terminal HVDC transmission operating in unbalanced condition. The controller was able to control directly the active power and reactive power toward some reference for each station in the inverter side, while the reference for the rectifiers is obtained with a PI controller that regulate the voltage in the DC link at the same time. The scheme included an adaptation law to handle the uncertainties in the parameters. The control was able to operated in unbalanced condition in any of the converters, the key for the design of the controller in unbalanced operation is based on a more complete description of the source voltage, that is both the positive and negative sequence were considered in the design controller of the DPC.

8. REFERENCES


