Distribution, coil-span and winding factors for PM machines with concentrated windings

S. E. Skaar, Ø. Krøvel, R. Nilssen

Abstract—Usage of concentrated windings in electrical machines is a well known topic, yet there has not been presented a direct approach on winding factor calculation. In this paper a method for directly calculating the winding factor, without doing a winding layout first, is proposed. A feasible region for the number of slots per pole per phase for machines with concentrated windings is also presented. Effects on the odd and even number of slots have on both the winding space harmonics and the air gap force is discussed.

Index Terms— axial flux machines, concentrated winding, permanent magnet, radial flux machines

I. INTRODUCTION

THE aim of this paper is to approach calculation of the winding factor for permanent magnet, PM, machines with concentrated windings at a different angle than previously proposed in [1]-[4]. The presented approach yields for both axial flux permanent magnet, AFPM, and radial flux permanent magnet, RFPM, machines with concentrated windings. The proposed method approaches the winding factor by calculating the distribution factor and coil-span factor, similar to the ones proposed by Gieras et al. [5] for three phase windings distributed in slots and Say [6] for fractional slot windings. The proposed approach does not require having an in-depth knowledge of the winding layout unlike the methods presented in [1]-[4]. Analytical expressions of factors: k_w, winding, k_m, distribution and k_e, coil-span will be presented. Comparison of different winding factors, spanning from fundamental to 49th winding space-harmonic, for different number of slots per pole per phase, q, will be presented. In addition FEA, Finite Element Analyse, calculations of resulting force fields of different slots and pole combinations will be presented.

S. E. Skaar was with the Norwegian University of Science and Technology, Dept. of Electrical Power Engineering, O. S. Bragstadsplass 2E, 7491 Trondheim, Norway. He is now with Smartmotor AS, Stiklestadveien 1, 7041 Trondheim, Norway (phone: +47 73 98 25 18; fax: +47 98 25 01; e-mail: skaar@smartmotor.no).

Ø. Krøvel is with the Norwegian University of Science and Technology, Dept. of Electrical Power Engineering, O. S. Bragstadsplass 2E, 7491 Trondheim, Norway (e-mail: oystein.krovel@elkraft.ntnu.no).

R. Nilssen is with the Norwegian University of Science and Technology, Dept. of Electrical Power Engineering, O. S. Bragstadsplass 2E, 7491 Trondheim, Norway (e-mail: robert.nilssen@elkraft.ntnu.no).

II. THEORETICAL EQUATIONS

A. Winding factor calculation

Number of slots per pole per phase is common to denote as q in electrical machine design. When q is larger than or equal to 1 the winding is called distributed, distributed windings can be divided into integer (q an integer) and fractional (q a fraction) windings. Machines designed with concentrated windings, where q<1 will always be fractional. They follow the same set of rules as fractional slot windings. q is found from::

$$q = \frac{N_s}{N_{ph} \cdot N_m} \qquad (1)$$

Where N_s is number of slots, N_m is number of poles and N_{ph} is number of phases. Being a fraction q can be expressed as:

$$q = \frac{z}{b} \qquad (2)$$

Here the numerator z is the N'_s/N_{ph} number of coils in a N'_m pole unit, expressed as denominator b, z is found from:

$$z = \frac{N_s}{\gcd(N_s, N_m \cdot N_{ph})} \qquad (3)$$

gcd is the greatest common divisor between the number of slots and the product of number of poles and number of phases.

(3) only applies for machines with all teeth wound with concentrated coils. Having concentrated coils wound around every second teeth or iron powder cores makes it more convenient to rephrase (3). Instead of expressing z in terms of the slot number, the number of coils, N_c, is introduced. Having a machine with all teeth wound will result in $N_c = N_s$, whereas a machine with alternate teeth wound would have $N_c = N_s/2$. Taking this into account (3) becomes:

$$z = \frac{N_c}{\gcd(N_c, N_m \cdot N_{ph})} \qquad (4)$$

In this work N_{ph}=3 with a phase spread, σ, equal to 60° is assumed, since this phase spread will ensure the best possible distribution factor for three phase machines.

1) Distribution factor

Distribution factor, where *n* is the winding space harmonic order and σ is phase spread angle equal to 60°, is found from:

$$k_{mn} = \frac{\sin(\frac{1}{2}n\sigma)}{z\sin(\frac{n\sigma}{2z})} \qquad (5)$$

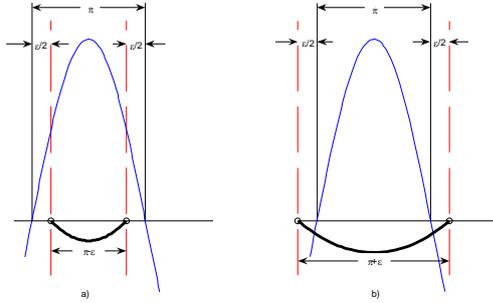This will basically be a rewrite of the way Say [6] presents the distribution factor for fractional slot windings.

Fig 1 Chording angle ε when a) coil span is shorter than pole pitch of π b) longer than pole pitch π

2) Coil-span factor

In order to find the coil-span factor, the slot pitch angle, γ_s, must be obtained:

$$\gamma_s = \frac{\pi \cdot N_m}{N_s} = \frac{\pi}{q \cdot N_{ph}} \qquad (6)$$

The chording or coil-span angle, ε, is:

$$\varepsilon = \pi - \gamma_s \qquad (7)$$

An illustration of the coil span is shown in Fig 1. The coil-span factor can be found from:

$$k_{en} = \cos(\frac{1}{2}n\varepsilon) \qquad (8)$$

3) Winding factor

The resulting winding factor is simply the product of the distribution and coil-span factor:
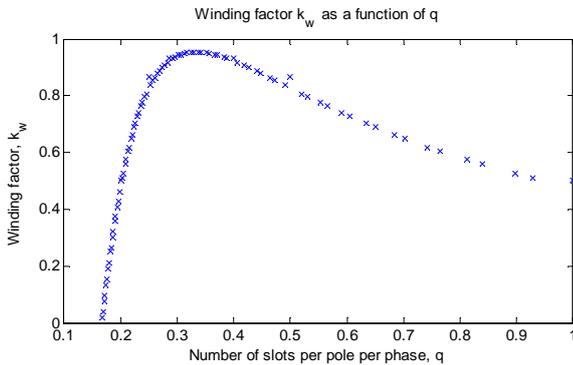
$$k_{wn} = k_{mn}k_{en} \qquad (9)$$

Fig 2 Fundamental winding factor k_w as a function of q

B. Feasible region of q for concentrated windings

Designing a machine with concentrated windings would seek to achieve the highest possible winding factor. In Fig 2 the winding factor variation with different values of q is plotted. It

is clear that the rapid decrease of the winding factor for q < ¼ makes this an unfeasible region. Although the decrease from q > ½ towards q = 1 is not that steep it is still an unfeasible region in ways of the poor winding factor found here.

From Fig 2 it can be seen that the best winding factor is obtained when q is close to $1/N_{ph}$ which yields a coil pitch of approximately π rad.
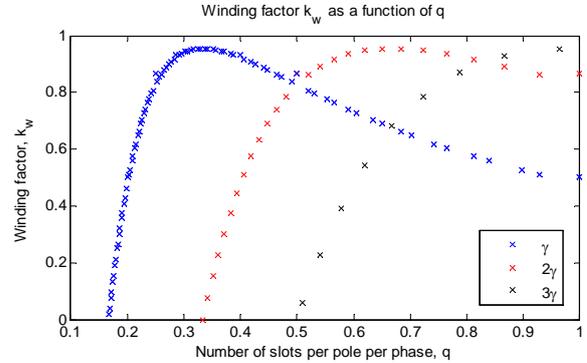
Fig 3 Fundamental winding factor k_w as a function of q and slot pitch

Table 1 show how the resulting coil pitch angle varies for different values of q. This is a highly hypothetical approach, since letting the coil pitch span over 2 or 3 slot pitches would result in a void space in the slots between. Fig 3 shows the winding factor variation of table 1. In a practical design the number of slots would have been decreased, hence giving a better winding factor, but also a q in the feasible region.

It is proposed to define the feasible region as:

$$q \in [\frac{1}{4}, \frac{1}{2}] \qquad (10)$$

The theoretical peak for the winding factor is $1/N_{ph}$, by moving the limits for the feasible region (10) towards this point will improve the achieved winding factor. For machines with a low pole and slot configuration this would be difficult to enforce, whereas machines with high pole and slot numbers, the freedom increases.

C. General design rules

When designing an electrical machine, certain rules upon pole and slot combinations exist. These rules also apply for concentrated windings and are listed as follows.

The number of poles has to fulfil three absolute requirements.
- First of all the pole number must be an even number.
- Further the number of pole pairs, P_p, in a section, F, of the machine can not be a multiple of the phase number, since this would lead to unbalanced windings [6]-[8].
- The final requirement is that the number of poles can not be equal to the number of slots, since this would lead to an undesired cogging torque in the machine in addition to the machine being a single phase machine.

The number of slots must be a multiple of N_{ph}.

The section F is found from:

$$F = \gcd(N_s, N_m / 2) \quad (11)$$

From this the number of pole pair in one section is:

$$P_p = \frac{N_m}{2F} \quad (12)$$

Number of slots per section is

$$N_a = \frac{N_s}{F} \quad (13)$$

Number of sections F also has the practical info, when modeling the machine in FEA, upon how large portion of the machine needs to be modeled in order to have a balance between both number of poles and number of slots. If F equals 1, the complete machine needs to be modeled, whereas if F equals 4 only a quarter of the machine is sufficient for the FEA model, and so on.

Applying these rules on machines with concentrated windings and having the number of phases equal to 3, the only possible number of slots is 3, 6, 9, 12,…, 3x, for machines with all teeth wound. For machines with alternate teeth wound this series would be reduced to only account for number of slots equal to 6, 12, 18, 24,…, 6x which actually omits any odd number of slots.

III. WINDING FACTORS TABLE

Tables 2 and 3 show the fundamental winding factor for machines with all teeth wound with concentrated coils. Both odd and even slot numbers are included. The gray regions are combinations outside the proposed feasible region. The pink winding factors are the region boarders, with q = ¼ and ½ respectively. Red areas are combinations left out by design rules and dark red are combinations where $N_s = N_m$. Tables 4 and 5 show winding factors for alternate wound teeth, the same color codes applies for these tables. For alternate teeth

Table 2 Fundamental winding factor, all teeth wound, $N_m = 2$ up to $N_m = 34$

Ns/Nm	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34
3	0.866	0.866	q<1/4														
6	q>1/2	0.866		0.866	q<1/4												
9		q>1/2	0.866	0.945	0.945	0.866	q<1/4										
12			q>1/2	0.866	0.933		0.933	0.866	q<1/4								
15				q>1/2	0.866		0.951	0.951		0.866	q<1/4						
18					q>1/2	0.866	0.902	0.945		0.945	0.902	0.866	q<1/4				
21						q>1/2	0.866	0.89		0.953	0.953		0.89	0.866	q<1/4		
24							q>1/2	0.866		0.933	0.949		0.949	0.933		0.866	q<1/4
27								q>1/2	0.866	0.877	0.915	0.945	0.954	0.954	0.945	0.915	0.877
30									q>1/2	0.866	0.874		0.936	0.951		0.951	0.936
33										q>1/2	0.866		0.903	0.928		0.954	0.954
36											q>1/2	0.866	0.867	0.902	0.933	0.945	0.953
39												q>1/2	0.866	0.863		0.917	0.936
42													q>1/2	0.866		0.89	0.913
45														q>1/2	0.866	0.858	0.886
48															q>1/2	0.866	0.857
51																q>1/2	0.866
54																	q>1/2

Table 3 Fundamental winding factor, all teeth wound, $N_m = 36$ up to $N_m = 68$

Ns/Nm	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68
27	0.866	q<1/4															
30		0.874	0.866	q<1/4													
33		0.928	0.903		0.866	q<1/4											
36		0.953	0.945	0.933	0.902	0.867	0.866	q<1/4									
39		0.954	0.954		0.936	0.917		0.863	0.866	q<1/4							
42		0.945	0.953		0.953	0.945		0.913	0.89		0.866	q<1/4					
45		0.927	0.945	0.951	0.955	0.955	0.951	0.945	0.927		0.886	0.858	0.866	q<1/4			
48		0.905	0.933		0.949	0.954		0.954	0.949		0.933	0.905		0.857	0.866	q<1/4	
51		0.88	0.901		0.933	0.944		0.955	0.955		0.944	0.933		0.901	0.88		0.866
54	0.866	0.854	0.877	0.902	0.915	0.93	0.945	0.949	0.954		0.954	0.949	0.945	0.93	0.915	0.902	0.877
57	q>1/2	0.866	0.852		0.894	0.912		0.937	0.946		0.955	0.955		0.946	0.937		0.912
60		q>1/2	0.866		0.874	0.892		0.933	0.936		0.951	0.954		0.954	0.951		0.936
63			q>1/2	0.866	0.85	0.871	0.89	0.905	0.919		0.945	0.948	0.953	0.955	0.955	0.953	0.948
66				q>1/2	0.866	0.849		0.887	0.903		0.928	0.938		0.951	0.954		0.954
69					q>1/2	0.866		0.867	0.884		0.913	0.925		0.943	0.949		0.955
72						q>1/2	0.866	0.847	0.867		0.902	0.911	0.933	0.933	0.945	0.949	0.953
75							q>1/2	0.866	0.846		0.88	0.895		0.92	0.93		0.945
78								q>1/2	0.866		0.863	0.879		0.906	0.917		0.936
81									q>1/2	0.866	0.845	0.862	0.877	0.891	0.904	0.915	0.925
84										q>1/2	0.866	0.844		0.875	0.89		0.913
87											q>1/2	0.866		0.859	0.874		0.899
90												q>1/2	0.866	0.843	0.858	0.874	0.886
93													q>1/2	0.866	0.843		0.871
96														q>1/2	0.866		0.857
99															q>1/2	0.866	0.842
102																q>1/2	0.866
105																	q>1/2

Table 4 Fundamental winding factor, alternate teeth wound, $N_m = 2$ up to $N_m = 34$

Ns/Nm	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34
6	q>1/2	0.866		0.866	q<1/4												
12			q>1/2	0.866	0.966		0.966	0.866	q<1/4								
18					q>1/2	0.866	0.902	0.945		0.945	0.902	0.866	q<1/4				
24							q>1/2	0.866		0.966	0.958		0.958	0.966		0.866	q<1/4
30									q>1/2	0.866	0.874		0.936	0.951		0.951	0.936
36											q>1/2	0.866	0.87	0.902	0.966	0.945	0.956
42													q>1/2	0.866		0.89	0.913
48															q>1/2	0.866	0.859
54																	q>1/2

Table 5 Fundamental winding factor, alternate teeth wound, $N_m = 36$ up to $N_m = 68$

Ns/Nm	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68
24		q<1/4															
30		0.874	0.866	q<1/4													
36		0.956	0.945	0.966	0.902	0.87	0.866	q<1/4									
42		0.945	0.953		0.953	0.945		0.913	0.89		0.866	q<1/4					
48		0.907	0.966		0.958	0.956		0.956	0.958		0.966	0.907		0.859	0.866	q<1/4	
54	0.866	0.854	0.877	0.902	0.915	0.93	0.945	0.949	0.954		0.954	0.949	0.945	0.93	0.915	0.902	0.877
60	q>1/2	0.866		0.874	0.893		0.966	0.936		0.951	0.955		0.955	0.951		0.936	
66				q>1/2	0.866	0.849		0.887	0.903		0.928	0.938		0.951	0.954		0.954
72						q>1/2	0.866	0.848	0.87		0.902	0.912	0.966	0.933	0.945	0.958	0.956
78									q>1/2	0.866	0.863	0.879		0.906	0.917		0.936
84										q>1/2	0.866	0.845			0.876	0.89	0.913
90												q>1/2	0.866	0.843	0.858	0.874	0.886
96														q>1/2	0.866		0.859
102																q>1/2	0.866
108																	q>1/2

wound it is not possible to have an odd number of slots.

The tables show that with $N_s > 30$ a winding factor $k_w > 0.94$ is achieved when choosing $N_m = N_s \pm 2$ or $N_m = N_s \pm 4$.

IV. WINDING SPACE HARMONICS

Different combinations of slot and pole number effects the winding space harmonics. These figures show that some slot and pole combinations reintroduce higher harmonic winding factors. It also shows symmetry for the harmonic contribution for even slot numbers. For odd slot numbers this symmetry needs a high number of harmonics to be revealed.
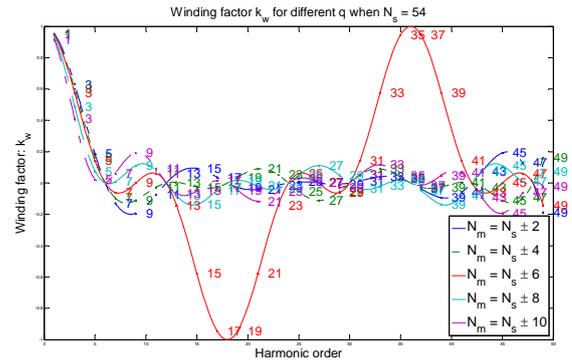
Fig 5 Winding factor, all teeth wound N_s=54
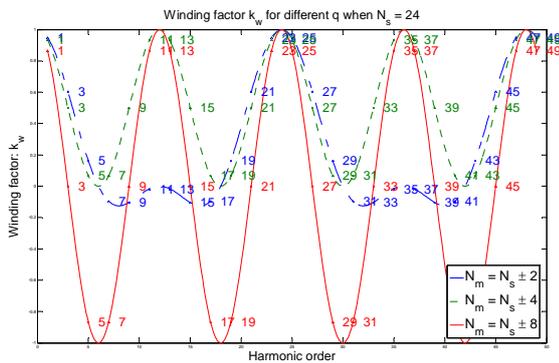
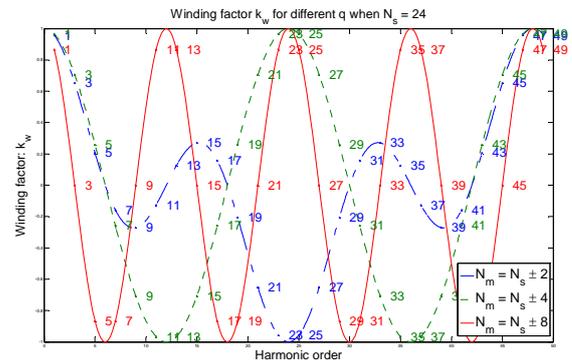Fig 4 Winding factor, all teeth wound N_s=24

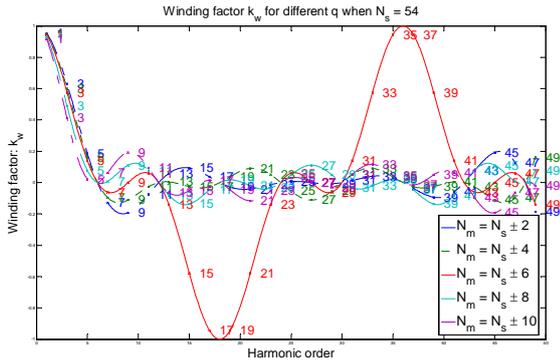Fig 6 Winding factor, alternate teeth wound N_s=24

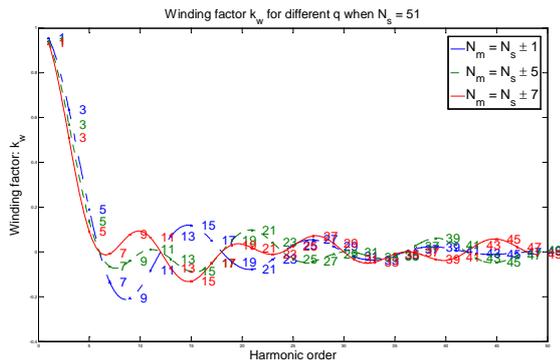Fig 7 Winding factor, alternate teeth wound $N_s = 54$

Fig 8 Winding factor, all teeth wound $N_s = 51$

V. ATTRACTIVE FORCES IN THE AIR GAP

As long as one uses iron as the flux carrier in electric machines there will be attractive forces between rotor and stator. It is a know fact that rotating attractive forces (radial for RFPM and axial for AFPM) exists in machines where q is not an integer [8]-[10]. Since machines with concentrated windings always have fractional type winding this is something to be aware of during the design of the machine. These sub harmonic force waves (i.e. force waves with longer wavelength than the pole width) will if the machine is badly designed create noise and vibrations. The important issue here is that the force wave, with its harmonics, should not excite any of the resonant frequency of the machine.

In Fig 9 the attractive force component around the air gap is plotted for machines with different pole-slot combinations. The force is found be integrating the density over the surface of each tooth at a given position. This means that information about higher harmonics is lost, and therefore only the fundamental for the different cases are investigated. The force values have been normalized with respect to the amplitude value of each case. From the figure it can be seen that the attractive forces have the same number of poles, or number of periods as the difference between poles and slots.

From Fig 9 it can be concluded that the fundamental wave length of the attractive force in radians is:

$$\tau_{F,1} = \frac{2\pi}{|N_s - N_m|} \qquad (14)$$
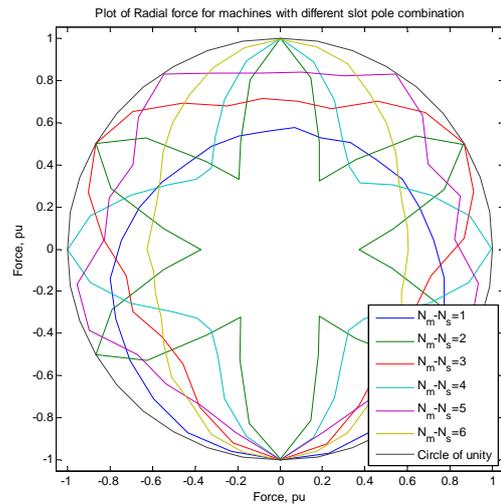
Fig 9 Plot of force distribution for different slot pole combinations

This fundamental force wave will rotate at a speed proportional the rotor speed, but in opposite direction:

$$\omega_{F,1} = \frac{-N_m}{|N_s - N_m|} \cdot \omega_r \qquad (15)$$

Usually the main concern in RFPM is whether the attractive force between rotor and stator are symmetric. This does not apply for the AFPM since the attractive forces are axial direction. Having an eccentric force in a RFPM will act much in the same way as having a mechanical unbalanced rotor. In Table 6 the total integrated force at a given instant (same as Fig 9) for radial direction is given. Considering the numerical accuracy of the model the radial forces for case 2-6 can be considered zero, i.e. balanced out or symmetrical. Case 1 ($|N_m-N_s|=1$) has a relatively high resulting radial force, the base value is the same as for Fig 9. The conclusion from Table 6 is that for all other than case 1 the attractive force between rotor and stator are symmetrical.

Table 6 Total integrated radial force

$|N_m-N_s|$	F_{rad} [pu]
1	3.35
2	0.05
3	0.04
4	0.05
5	0.06
6	0.05

As mentioned the important issue is whether the different force waves excites resonant frequencies in the machine. Using the radial flux machine as an example it has several different modi for its resonances (Fig 10). The lowest corresponds to the even push and pull between rotor and stator (DC level), the next is the eccentric pull (one period), then the elliptic shaped (two periods) etc. From Fig 9 on can see that this resembles the shape of the different curves for the

attraction forces. They all have a DC-component (Mode 0), but they all have different fundamental, and therefore will excite different resonance frequencies in the machine. $|N_m-N_s|=1$ is eccentric (Mode 1), $|N_m-N_s|=2$ is elliptic (Mode 2), $|N_m-N_s|=$ is triangular (Mode 3) etc. Based on this knowledge, a pole slot combination can be chosen so that its fundamental force wave does not excite a critical mode in a given machine. It is obvious that the amplitude value and the shape are depending on the geometry of the air gap, but this has not been considered in this work.
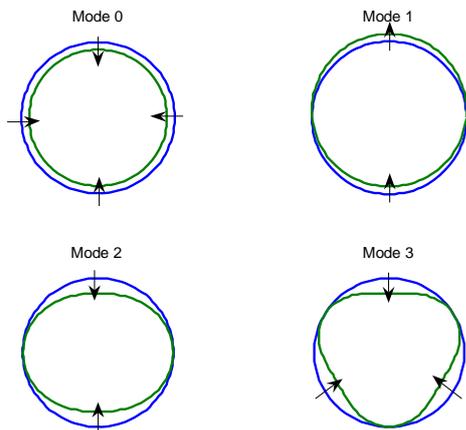
Fig 10 Four different modus of resonance in the radial directions

VI. CONCLUSION

A method of calculating winding factors for concentrated coils without having knowledge of the winding layout has been proposed. The method is based on a fractional slot approach used for distributed windings. The proposed method yields for concentrated windings on both machines with all and alternate teeth wound. The winding factor variation for different values of q has been shown, and based on this a feasible region for q has been proposed to [¼, ½].

Tables of winding factors for different slot and pole combinations have been presented both for machines with all and alternate teeth wound. The tables show that choosing $N_s>30$ and $N_m = N_s \pm 2$ or $N_m = N_s \pm 4$ will always ensure a winding factor $k_w > 0.94$, for any allowed slot and pole combination when dealing with concentrated windings.

The winding space harmonic variation for different slot and pole combinations has been presented; where it was shown how higher order space harmonics may be reintroduced for some combinations.

The force calculations show that for all other combinations than Ns=Nm ± 1 yields symmetric attractive forces in RFPM. For both AFPM and RFPM one should be aware of the relationship between resonance frequencies of the machine and the wave length of the fundamental force wave.

REFERENCES

[1] Cors J., Viarouge P., "Synthesis of high performance pm motors with concentrated windings", *IEEE Transaction on Energy Conversion*, Vol. 17 Issue 2, 2002, pp. 248-253

[2] Libert F., Soulard J., "Investigation on Pole-Slot Combinations for Permanent-Magnet Machines with Concentrated Windings", *International Conference Electrical Machines (ICEM 04)*, 5.-8. September 2004, Cracow, Poland

[3] Libert F., Design, "Optimization and Comparison of Permanent Magnet Motors for a Low-Speed Direct-Driven Mixer", *Licentiate Thesis*, 2004, KTH, Stockholm, Sweden

[4] Ishak D., Zhu Z. Q., Howe D., "Comparison of PM Brushless Motors, Having Either All Teeth or Alternate Teeth Wound", *IEEE Transactions on Energy Conversion*, 2005

[5] Gieras, J. F., Wang, R.-J., Kamper, M. J., Axial Flux Permanent Magnet Brushless Machines, Kluwer Academic Publishers, The Netherlands, 2004

[6] Say M. G., "The Performance and Design of Alternating Current Machines", 2nd Edition, Pitman's, London, 1948

[7] Walker, J. H., Large Synchronous Machines, Design, manufacture, and operation, Oxford University Press, USA, 1981

[8] Taraldsen, B., Statorschwingungen in synchronmaschinen mit bruchlochwicklungen, PhD Thesis, NTH, Norway, 1985

[9] Hanselmann, D.C., Dr., Brushless Permanent Magnet Motor Design. Second ed. 2003: The Writers' Collective, Cranston, Rhode Island. 392.

[10] Magnussen, F., On Design and Analysis of Synchronous Permanent Magnet Machines for Field-weakening Operation in Hybrid Electric Vehicles, in Electrical Machines and Power Electronics, Department of Electrical Engineering. 2004, Royal Institute of Technology: Stockholm. p. 251.