Analytical Calculations of AC Side Harmonic Losses in Three-Level Converters

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Abstract
This paper is presenting analytical expressions for the AC side harmonic losses in 3-level converters. Verification is done by comparison to circuit simulations and experimental results from a test setup. A comparison of AC side harmonic losses between 2- and 3-level converters is also given.

Introduction
Three-level converters are often used for large motor drives. In this application, it may be useful to have a tool for calculating the ripple current on the AC-side of the converter. This paper is based upon previous work done in [1]. In [1] a method to calculate the rms value of the ripple current on the AC side of a two-level converter is presented. This paper uses this method for three-level converters and presents equations for calculating the rms value of the ripple current on the AC-side, and the method to derive these equations on three-level converters.

The modulation method used is simple sinusoidal modulation, but it is possible to derive equations from other continuous modulating functions as well. The modulation functions used are:

\[
\alpha_a = \frac{1}{2} \left( 1 + M \cos(\omega t) \right), \quad \alpha_b = \frac{1}{2} \left( 1 + M \cos(\omega t - \frac{2\pi}{3}) \right), \quad \alpha_c = \frac{1}{2} \left( 1 + M \cos(\omega t + \frac{2\pi}{3}) \right)
\]

Equivalent scheme
The derivation of the ripple current is performed with the equivalent scheme shown in Fig 1. An inductor and a voltage source is wye connected to each phase of the three-level converter. \( u_O \) is the state space vector form the converter, \( u_U \) is the continuous travelling space vector from the voltage sources, and \( \Delta i_N \) is the ripple current.

\[ u_O \]
\[ u_U \]
\[ \Delta i_N \]

Fig 1: Equivalent scheme for the converter output voltage and the load.
Symmetry and space vectors

Because of symmetry, it is sufficient to analyse a $60^0$ interval as seen in Fig 2.

![Diagram showing output space vectors in a $60^0$ interval.]

The output state space vectors from the converter can be expressed as [1]:

$$
\mathbf{u}_O = \frac{2}{3} \left( \mathbf{u}_a + \bar{a} \mathbf{u}_b + \bar{a}^2 \mathbf{u}_c \right)
$$

$$
\bar{a} = \left( -\frac{1}{2} + j \frac{\sqrt{3}}{2} \right)
$$

The total space vector is a function of the duty ratio of each bridge leg and the space vectors corresponding to the apexes of the triangle. The function is depending on which area the travelling space vector is in. For $\mathbf{u}_O$ in area 2:

$$
\mathbf{u}_{O,2} = d_{00-} \cdot \mathbf{u}_{00-} + d_{0+-} \cdot \mathbf{u}_{0+-} + d_{++-} \cdot \mathbf{u}_{++-} + d_{++0} \cdot \mathbf{u}_{++0}
$$

$d_{aan}$ is the duty ratio of each voltage vector within a switching interval.

To analyse the ripple current, the switching intervals must be analysed for every area in Fig 2. This is done in [2] and presented in Table I.

Table I: Switching sequences.

<table>
<thead>
<tr>
<th>Area</th>
<th>Angle</th>
<th>$t_\mu = 0$</th>
<th>Switching sequence</th>
<th>$t_\mu = Tp/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\pi/3$ to $\pi/2$</td>
<td>00-</td>
<td>000</td>
<td>0+0</td>
</tr>
<tr>
<td>1</td>
<td>$\pi/2$ to $2\pi/3$</td>
<td>-0-</td>
<td>00-</td>
<td>000</td>
</tr>
<tr>
<td>2</td>
<td>$\pi/3$ to $\phi(M)$</td>
<td>00-</td>
<td>0+-</td>
<td>++-</td>
</tr>
<tr>
<td>3</td>
<td>$\phi(M)$ to $\pi/2$</td>
<td>00-</td>
<td>0+-</td>
<td>0+0</td>
</tr>
<tr>
<td>3</td>
<td>$\pi/2$ to $\phi(M)$</td>
<td>-0-</td>
<td>00-</td>
<td>0+-</td>
</tr>
<tr>
<td>4</td>
<td>$\phi(M)$ to $2\pi/3$</td>
<td>-0-</td>
<td>++-</td>
<td>0+0</td>
</tr>
</tbody>
</table>

Example: 0+- means that bridge leg $a$ is connected to the neutral point, bridge leg $b$ and bridge leg $c$ are connected to the positive and negative dc-bus respectively.
Method of derivation

There exist three different calculations depending on the modulation depth, M. The three sectors are depicted in Fig 2. For \( M < \frac{1}{\sqrt{3}} \) (innermost sector) the three-level converter behaves almost like a two-level converter and the equation for the ripple current will be similar to a two-level converter.

From the equivalent scheme, the ripple current can be derived from [1]:

\[
\frac{d\Delta i_{N\mu}}{dt_{\mu}} = \frac{1}{L} \left[ u_{00} - u_{00}\tau(\tau) \right]
\]

(4)

\( t_{\mu} \) is the microscopic time within a switching interval, between 0 and \( T_p \). The following derivation of currents must be done in every area in Fig 2, but below are listed the ripple currents in area 2.

\[
\begin{align*}
\Delta i_{N\mu 1}(\tau) &= d_{00-} \left[ u_{00-} - u_{00,2}(\tau) \right] \frac{T_p}{2L} \\
\Delta i_{N\mu 2}(\tau) &= \Delta i_{N\mu 1}(\tau) + d_{00+} \left[ u_{00+} - u_{00,2}(\tau) \right] \frac{T_p}{2L} \\
\Delta i_{N\mu 3}(\tau) &= \Delta i_{N\mu 1}(\tau) + d_{+0+} \left[ u_{+0+} - u_{00,2}(\tau) \right] \frac{T_p}{2L} = 0
\end{align*}
\]

(5)

are decomposed into the stator oriented coordinate system by:

\[
\begin{align*}
\Delta i_{N\mu 1,\alpha}(\tau) &= \text{Re}\left( \Delta i_{N\mu 1}(\tau) \right) \\
\Delta i_{N\mu 1,\beta}(\tau) &= \text{Im}\left( \Delta i_{N\mu 1}(\tau) \right)
\end{align*}
\]

(6)

The sum of the squares of the phase currents can be transformed to the stator oriented coordinate system \( \alpha \) and \( \beta \) [1].

\[
\Delta i_{N,\alpha}^2 + \Delta i_{N,\beta}^2 = \frac{3}{2} \left[ \Delta i_{N,\alpha}^2 + \Delta i_{N,\beta}^2 \right] = \frac{3}{2} \left| \Delta i_{N} \right|^2
\]

(7)

The microscopic derivation of the ripple current within a switching interval is:

\[
\begin{align*}
&= \frac{1}{t_{\mu,j+1} - t_{\mu,j}} \int_{t_{\mu,j}}^{t_{\mu,j+1}} \left[ \Delta i_{N\alpha}^2(t_{\mu}) + \Delta i_{N\beta}^2(t_{\mu}) \right] dt_{\mu} \\
&= \frac{1}{3} \left[ (\Delta i_{N,\alpha,j}^2 + \Delta i_{N,\beta,j}^2 + \Delta i_{N,\alpha,j}^2) + (\Delta i_{N,\beta,j+1}^2 + \Delta i_{N,\alpha,j+1}^2 + \Delta i_{N,\beta,j+1}^2) \right]
\end{align*}
\]

(8)

The local rms values (related to half a switching interval) of the current ripple can be expressed as (9) shows. Again the equation for area 2 is listed.

\[
\Delta i_{N,\alpha,\beta,\mu,j}^2(\tau) = \frac{2}{T_p} \int_{0}^{T_p/2} \left[ \Delta i_{N,\alpha}^2(t_{\mu}) + \Delta i_{N,\beta}^2(t_{\mu}) \right] dt_{\mu}
\]

(9)

To derive the total harmonic rms current per phase, the local rms currents must be integrated over the entire 60° interval and divided by 3:
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\[ \Delta I_{N,\text{rms}}^2 = \frac{1}{3} \frac{2\pi/3}{\pi/3} \int \Delta I_{N,abc,\text{rms}}^2 d\alpha \]  

(10)

Hence, the equations of the rms current on the AC-side are listed below. They are dependent on the modulation depth.

\[ M = \left[ \frac{2}{3} \text{ to } \frac{2}{\sqrt{3}} \right], \text{ (uttermost sector)} \]

\[ \Delta I_{N,\text{rms}}^2 = \Delta I_n^2 \left( \frac{1}{54} \left( -48\sqrt{3}M^3 - 24\sqrt{3}M^3 \cos(\varphi_2) - 108M \sin(\varphi_2) - 24\varphi_2 \right) 
+ 108M^2 \varphi_1 + 36\sqrt{3}M^2 - 36M^3 - 81M^3 \sin(\varphi_1) - 108M \sin(\varphi_1) - 24\varphi_1 \right. 
+ 54M^2 \sin(\varphi_1) \cos(\varphi_1) - 54\sqrt{3}M^2 \cos(\varphi_1) - 36 \sqrt{3}M \cos(\varphi_1) 
\left. + 9\sqrt{3}M^3 \cos(\varphi_1) + 24\sqrt{3}M^3 \cos(\varphi_1) - 108M^2 \varphi_2 - 108M \sin(\varphi_2) - 24\varphi_2 \right) \]  

(11)

\[ M = \left[ \frac{1}{\sqrt{3}} \text{ to } \frac{2}{3} \right], \text{ (middle sector)} \]

\[ \Delta I_{N,\text{rms}}^2 = \Delta I_n^2 \left( \frac{1}{18} \left( -16\sqrt{3}M^3 - 8\sqrt{3}M^3 \cos(\varphi_3) - 8\varphi_3 \right) 
- 6\sqrt{3}M^3 + 36M^2 \sin(\varphi_3) \cos(\varphi_3) - 24\sqrt{3}M \cos(\varphi_3) - 24\sqrt{3}M^3 \cos(\varphi_3) 
+ 8\sqrt{3}M^3 \cos(\varphi_3) + 36M^2 \varphi_2 + 8M^2 \pi + 9M^4 \pi + 24\sqrt{3}M \cos(\varphi_3) 
+ 24\sqrt{3}M^3 \cos(\varphi_3) - 36M^2 \sin(\varphi_3) \cos(\varphi_3) - 12M^3 \right) / \pi \]  

(12)

\[ M < \frac{1}{\sqrt{3}}, \text{ (innermost sector)} \]

\[ \Delta I_{N,\text{rms}}^2 = \Delta I_n^2 \left( \frac{1}{18} \left( M^2 (9\pi M^2 - 12M - 6\sqrt{3} + 8\pi - 16\sqrt{3}M) \right) \right) \]  

(13)

Where

\[ \Delta I_n = \frac{U_{ac}T}{8L} \]  

(14)

By eliminating the angle expressions for (11) and (12), a simplified expression of the ripple current can be derived. Letting \( \varphi_1 = \varphi_2 = \pi/3 \) and \( \varphi_3 = 2\pi/3 \) in (11) and (12), we get a simplified common equation for the modulation range from \( M = \frac{1}{\sqrt{3}} \) to \( M = \frac{2}{\sqrt{3}} \).

\[ M=[\frac{1}{\sqrt{3}} \text{ to } \frac{2}{\sqrt{3}}] \]

\[ \Delta I_{N,\text{rms}}^2 = \Delta I_n^2 \left( \frac{4}{27} \pi - \frac{2}{3}M^3 + \frac{10}{9} \pi M^2 - \frac{8}{3} \sqrt{3}M + \frac{1}{2} \pi M^4 \right) \]  

(15)
Transfer between areas

The angles in the equations above describe the crossing between the areas in Fig 2. The crossing between the areas is derived in [2] and the results are repeated here.

\[ M = [2/3 \text{ to } 2/\sqrt{3}] \]

\[ \phi_1(M) = \pi - \phi_2(M) \]

\[ \phi_2(M) = \pi + \tan\left( \frac{1}{3} \left( \frac{1 + \sqrt{3} + 9M^2}{1 - \sqrt{3} + 9M^2} \right) \right) \]  

(16) (17)

\[ M = [1/\sqrt{3} \text{ to } 2/3] \]

\[ \phi_3(M) = \pi - \phi(M) \]

\[ \phi_4(M) = \frac{5\pi}{6} + \tan\left( \frac{2}{3} \left( \frac{2}{1} \left( -\frac{1}{3} \sqrt{1 + 3M^2} \right) \right) \right) \]  

(18) (19)

These angle expressions are made for solvers that returns a negative value of the \( \tan() \) expression.

Comparison of two- and three level converters

The ripple current in a two-level and a three-level converter are compared with the equations derived in this paper and [1]. The comparison is done for simple sinusoidal modulation in both cases.

Fig 3 shows that the ripple current of the three-level converter is less then half of the two level for M>0.5.
Verification

To verify the results, a test setup as shown in Fig 4 was used.

![Laboratory setup diagram](image)

Fig 4: Laboratory setup.

The primary data for the setup is summarized in Table II.

<table>
<thead>
<tr>
<th>Component</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPC Converter</td>
<td>20kW, $f_{sw}=1$kHz, 300V$_{dc}$</td>
</tr>
<tr>
<td>Induction machine</td>
<td>15kW, 230V, $L_{e}=3.2$mH</td>
</tr>
<tr>
<td>Load</td>
<td>15kW DC-machine with variable load</td>
</tr>
<tr>
<td>DSP/Control board</td>
<td>TMS320F2812 based</td>
</tr>
</tbody>
</table>

Table II: Data on laboratory setup.

In Fig 5, a plot of the output waveforms is shown at nearly full load and speed. The output current is analysed for different load conditions using Matlab and then compared to the analytical expression developed.

![Output waveforms](image)

Fig 5: Output waveforms at $M=0.98$. Trace1: Pole voltage [200V/div]. Trace 2: Phase current [33.3A/div].

The theory is also evaluated in a circuit simulation program. A three-level converter with a star connected inductor and a voltage source load is simulated in a circuit simulation program called Krean. The simulations and the experimental results are compared to the analytical expressions in Fig 6.
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**Fig 6:** Results from the analytical calculation, the experimental setup, and the Krean simulation.

As seen from Fig 6, the analytical solutions agree with the simulations and the experimental results fairly well. The error between the analytical solution given in (11)-(13), and the simplified equation becomes large when $M>0.95$ (error>10%).

**Conclusion**

Analytical expressions for the ripple current on the AC-side of a three-level converter are developed. By this, time consuming circuit simulations are avoided. A comparison of the ripple current with respect to the 2-level topology is also presented. Experimental measurements on a three-level prototype in the lab and simulations are done to verify the analytical expressions.

**References**
