Abstract—Sensorless control of PMSM at low and zero speed requires injection of a test signal to extract information on rotor position. Two methods are commonly used for this excitation; the INFORM method and injection of a high frequency carrier signal. This paper describes the comparison of the two types excitation signals applied to a prototype axial flux PMSM. The paper gives an evaluation of what excitation that would be preferred for the specified prototype machine.

Index Terms—Sensorless Control, PMSM, INFORM.

I. INTRODUCTION

Sensorless control of electric machines has been a research topic for more than 10 years. The goal for this work is to eliminate the need for a position sensor by using the machine as the position sensor. This task has been given a lot of attention since the position sensor and cable connection has been a source of unreliability for motor control applications. Sensorless control of a PMSM is divided into two speed regions; a high speed region and a low speed region. In the high speed region the back EMF can be used for the rotor position estimate. In the low and zero speed region he back EMF becomes too small for position estimation. In this region some extra excitation is required to get a rotor position estimate. There are two excitation methods that have been successful in extracting this information; injection of high frequency sinusoidal signal and the INFORM method. The methods use the same phenomena; the electromagnetic saliency present in the machine. There are several differences in these two methods, the ability to extract the saliency in a prototype axial flux machine for the two excitation methods is evaluated in this paper.

II. MACHINE MODEL

A PMSM machine model is easiest represented in the rotating dq reference system. Different machine designs offer different type of saliencies in the machine. IPMSMs offer saliency due to inherent different permeabilities in the d and q–axes. Surface mounted PMSMs may have saliency due to saturation, or the design may be without measurable saliency. The presence of a single saliency in a PMSM results in a different inductance in the dq axis. The voltage equations for the stator winding in the dq reference system are:

\[
\begin{bmatrix}
    u_d \\
    u_q
\end{bmatrix} =
\begin{bmatrix}
    r_s & 0 \\
    0 & r_s
\end{bmatrix}
\begin{bmatrix}
    i_d \\
    i_q
\end{bmatrix} +
\begin{bmatrix}
    s & \omega \\
    - \omega & s
\end{bmatrix}
\begin{bmatrix}
    \psi_d \\
    \psi_q
\end{bmatrix}
\]

The flux linkages are defined as:

\[
\begin{bmatrix}
    \psi_d \\
    \psi_q
\end{bmatrix} =
\begin{bmatrix}
    L_q & 0 \\
    0 & L_d
\end{bmatrix}
\begin{bmatrix}
    i_d \\
    i_q
\end{bmatrix} +
\begin{bmatrix}
    \psi_d \\
    \psi_q
\end{bmatrix}
\]

All equations are given in normalized form, and the time is also normalized. The machines used in this work have a single saliency present, thus

\[
L_d \neq L_q
\]

Fig. 1 Definition of axes reference systems
III. ESTIMATION OF ROTOR POSITION AT LOW SPEED

Estimation of rotor position at low and zero speed requires injection of a test signal. Two principal different excitation signals are evaluated by considering the INFORM method and injection of a high frequency carrier signal.

A. INFORM method

The INFORM method [1] is based upon online measurements of the inductance of the machine. There are several ways of implementing the measurement of the inductance. The basic method is based on measuring the inductance in each direction of the stator windings. Two voltage vectors are used in each direction to minimize the deviation of the fundamental current vector location.

![Diagram of stator reference frame with vectors](image)

**Fig.2 Measure vectors in stator reference frame**

To simplify the discussion we neglect the rotational induced voltage. The voltage equations then become:

\[
\begin{bmatrix}
    u_d \\
    u_q
\end{bmatrix} =
\begin{bmatrix}
    L_d & \frac{di_d}{dt} \\
    L_q & \frac{di_q}{dt}
\end{bmatrix}
\begin{bmatrix}
    di_d \\
    di_q
\end{bmatrix}
\]

(4)

Transforming the applied stator voltage to the rotating dq reference system:

\[
\begin{align*}
    |u_d| \cos(\gamma - \theta) &= L_s \frac{di_d}{dt} \\
    |u_q| \sin(\gamma - \theta) &= L_s \frac{di_q}{dt}
\end{align*}
\]

(5)

To get measurable quantities one substitutes for the derivative of the currents in αβ system:

\[
\begin{align*}
    \frac{di_d}{dt} &= \frac{di_d}{dt} \cos(\theta) - \frac{di_q}{dt} \sin(\theta) \\
    \frac{di_q}{dt} &= \frac{|u_e|}{L_s L_q} (L_s \cos(\gamma - \theta) \cos(\theta) - L_s \sin(\gamma - \theta) \sin(\theta)) \\
    \frac{di_d}{dt} &= \frac{di_d}{dt} \sin(\theta) + \frac{di_q}{dt} \cos(\theta) \\
    \frac{di_q}{dt} &= \frac{|u_e|}{L_s L_q} (L_s \cos(\gamma - \theta) \sin(\theta) - L_s \sin(\gamma - \theta) \cos(\theta))
\end{align*}
\]

(6) (7)

Figure 3 shows \(\frac{di}{dt}(\gamma)\). The rotor angle \(\theta\) is held constant.

\[
\frac{di}{dt} = \frac{di_a}{dt} + j \frac{di_b}{dt}
\]

(8)

![Graph showing length of current derivative vector as a function of voltage angle](image)

**Fig.3 Length of current derivative vector as a function of voltage angle with constant rotor position.**

In figure 3 \(\theta = 30^\circ\), \(L_d = 0.47\ pu\), \(L_q = 1.37\ pu\) and the applied voltage \(|u_e| = 0.5\ pu\)

By measuring the change in phase currents in the three directions indicated in figure 2, and using a vector summation of the three measurements, a complex vector \(\tau\) gives the double rotor position.

\[
\tau = \Delta ia + \Delta ib \cdot e^{j\gamma} + \Delta ic \cdot e^{-j\gamma}
\]

(9)

\[
2\theta = -\arg(\tau)
\]

(10)

A saturation test can be used to determine the positive d-axis [1].
In order to compare the two excitation methods the effective measuring frequency (or carrier signal frequency) is evaluated. The test setup used in this paper uses a 10 kHz switching frequency. In order to describe the test signal frequency a full duty cycle is used for each test vector. Figure 4 shows the test signal in phase A.

\[
\begin{align*}
\begin{bmatrix} u_a \\ u_b \end{bmatrix} &= \begin{bmatrix} r_s & 0 \\ 0 & r_s \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} + \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \begin{bmatrix} \psi_a \\ \psi_b \end{bmatrix}
\end{align*}
\]

The flux linkages are defined as:

\[
\begin{align*}
\psi_a &= [L - \Delta L \cos(2\theta) \ -\Delta L \sin(2\theta)] \begin{bmatrix} i_a \\ \psi_a \cos(\theta) \end{bmatrix} + [\psi_a \sin(\theta)] \\
\psi_b &= [-\Delta L \sin(2\theta) \ + L + \Delta L \cos(2\theta)] \begin{bmatrix} i_b \\ \psi_b \cos(\theta) \end{bmatrix} + [\psi_b \sin(\theta)]
\end{align*}
\]

The average and difference inductance are defined as:

\[
\begin{align*}
L &= \frac{L_a + L_d}{2} \\
\Delta L &= \frac{L_a - L_d}{2}
\end{align*}
\]

If we consider only the current component from the carrier signal and neglect the influence from the stator resistance (\(r_s\) small compared to \(\omega L\)) we get:

\[
U_{\text{e,eq}} = j \cdot \omega_0 \cdot L \cdot I_{\text{e,eq}}
\]

The current components in equation (17) form two rotating vectors; one rotating in positive direction and, one rotating in the negative direction.

\[
\begin{align*}
I_{\text{eq},c} &= \begin{bmatrix} I_{a,c} \\ I_{b,c} \end{bmatrix} = \begin{bmatrix} \cos(\omega_0 t) \\ \sin(\omega_0 t) \end{bmatrix} + I_{\text{e},c} \begin{bmatrix} \cos(-\omega_0 t + 2\theta) \\ \sin(-\omega_0 t + 2\theta) \end{bmatrix}
\end{align*}
\]

where:

\[
\begin{align*}
I_{\text{e},c} &= \frac{V_{e,\text{eq}}}{\omega_0} \frac{L}{L - \Delta L} \\
I_{\text{eq},c} &= \frac{V_{e,\text{eq}}}{\omega_0} \frac{\Delta L}{L - \Delta L}
\end{align*}
\]

The voltage equations in stationary reference frame are:

Fig.4 Test signal in phase A

The first harmonic of the applied voltage represents the effective frequency of the test signal. For this specific test signal the effective frequency is 5 kHz.

There are several ways of implementing the INFORM method. Different strategies are used to reduce the time consumed performing the online measurement of the inductance. The least time consuming method is to use only two measurements [1]. Integrating the INFORM measurement in the traditional PWM pattern [7] is also a very attractive way of implementing the INFORM measurement. In terms of effective measuring frequency the above mentioned approaches give a measuring frequency equal to or higher than twice the switching frequency.

### B. Injection of high frequency Carrier signal

Injection of a high frequency signal is a different approach to get an estimate of the rotor position [2]-[6]. The method is based on the same principle; use of saliency in the machine.

The voltage equations in stationary reference frame are:
signal is rotated back to its original rotating frame. The resulting signal would theoretically be:

\[ I'_{e} = I'_{e,n} \begin{bmatrix} \sin(2\theta - \omega t) \\ \cos(2\theta - \omega t) \end{bmatrix} \tag{20} \]

The resulting current in equation (20) contains spatial information of the saliency in the machine. This saliency is rotating at double frequency, relative to the carrier frequency, as a result of the bipolar nature of the saliency. The heterodyning process consists of a cross vector product that produces an error signal to a controller. Considering the cross product of vectors, in polar form, results in the multiplication of the two magnitudes and the sine of the angular difference of the two vectors. The saliency in the machine is represented by a unit vector rotating with a double frequency with respect to the carrier frequency.

The unit saliency vector can be written as:

\[ \hat{I}_e = \begin{bmatrix} \sin(2\theta - \omega t) \\ \cos(2\theta - \omega t) \end{bmatrix} \tag{21} \]

The heterodyning process uses equations (20) and (21), with error signal \( \varepsilon \) equal to:

\[
\varepsilon = \hat{I}_e \times I'_e \\
\varepsilon = I_{e,n}\sin(2\theta - \omega t)\cos(2\theta - \omega t) - I_{e,n}\cos(2\theta - \omega t)\sin(2\theta - \omega t) \\
\varepsilon = I_{e,n}\sin\left(2(\theta - \hat{\theta})\right) \tag{22}
\]

as \( \hat{\theta} \) approaches \( \theta \) the error signal \( \varepsilon \) becomes:

\[
\varepsilon = 2I_{e,n}(\theta - \hat{\theta}) \tag{23}
\]

The error signal in equation (23) gives a linear input to the controller in figure 5. This allows the observer controller to align the estimated unit vector to the measured current vector.

IV. EXPERIMENTAL WORK

The experimental work was done on an prototype axial flux PMSM. Figure 6 gives the principal design (number of coils and magnets are fictive).

![Fig.6 Principle of the machine design](image)

The machine ratings were 300 W, 24 V. The stator consists of concentrated wound coils, with an iron powder coil (fig.6a). The rotor consists of PM magnets and back iron. Figure 7 shows a cross section (principal) of the machine in an axial view.

![Fig.7 Axial cross-section of the machine](image)

The permanent magnets have relative permeability equal to 1.03 and the conductivity is comparable to conductivity in iron. The machine model would have \( L_{d} = L_{q} \) if there was no saturation present in the machine. There are two regions where saturation is most likely to occur. The first region is in the iron powder cores in stator. Second, there may be saturation in the rotor yoke, especially between the magnets.

The machine was connected to a drive system with a programmable DSP (TMS320c243). A balanced tree phase voltage was applied to the stator windings.

Saliency in the machine can be detected by representing the current response to the carrier signal in the stationary \( \alpha\beta \) frame. The frequency was adjusted and the current was held constant by adjusting the duty cycle. The machine was mounted on a turntable enabling measurements at different angles with locked rotor. Figure 8 shows the \( \alpha\beta \) currents when the rotor was turned 40 electrical degrees. The magnitude of the stator current was 6 A.
Figure 8 $\alpha$ vs. $\beta$ currents in stationary reference frame, carrier frequency equal 125 Hz

A circle (plotted with +) is drawn in the center of the current plot to highlight the ellipse shape of the current plot. The ellipse shapes indicate that a single saliency is present.

The balanced carrier signal was injected with several different frequencies. The current magnitude was held constant during the variation of the frequency. As the frequency increased the ellipse shaped $\alpha\beta$ currents turned into a circular shape. At 250 Hz carrier frequency the $\alpha\beta$ currents formed a circle indicating no saliency present.

Figure 9 $\alpha$ vs. $\beta$ currents, carrier frequency equal 250 Hz

The INFORM method was tried on the same machine. Different current levels were tried up to rated current for the machine. The current magnitudes where equal in all three voltage directions (fig.2), leaving no information on the rotor position.

V. COMPARISON OF THE EXCITATION SIGNAL

For a setup with switching frequency equal to 10 kHz the INFORM method will measure effective frequency equal to or higher than 5 kHz. The method based on injection of a high frequency voltage offers more freedom in the choice of measure frequency. The lower bound is given from the frequency separation needed to filter out the high frequency signal from the fundamental current [5]. One typically needs one decade separation between carrier signal and fundamental excitation. The upper bound is given by the ability to produce a high resolution in the modulation of the carrier signal.

VI. CONCLUSION

Measurements on the axial flux PMSM prototype machine showed that there was some saturation induced saliency present in the machine. There are two regions in the machine where one would expect some saturation; the iron powder core the in stator and the yoke in the rotor (fig.6). Given the frequency dependency in the saliency measurement one would expect that the saliency seen in the current plots is due to saturation in the rotor yoke. Given that the permanent magnets have the same conductivity as iron, one would expect circulating currents in the magnets. These circulating currents give a field that opposes the carrier signal flux. As the circulating currents will increase with the carrier frequency there will eventually be very little of the carrier flux penetrating into the rotor yoke.

Based on the experimental measurement one can conclude that the best candidate excitation for the prototype axial flux machine is the injection of a balanced three phase carrier signal. The low carrier signal frequency needed for this machine will give limitations on the range of the method as the rated frequency is 50 Hz. A combination of the high frequency injection and a back EMF method could give a drive solution for all speeds including zero speed.

REFERENCES